

ESM 206B
Hampton
April 23, 2009

Homework 3 – ANCOVA exercise

Puritan Oats Inc., a fictitious firm, recently conducted a research study. The subjects were selected by Puritan employees from a long list of Cheesetown, Wisconsin residents who wanted to participate.

Fifty-four female subjects between the ages of 21 and 40 were selected, supposedly at random. Twenty-seven were instructed to eat Puritan oatmeal each morning for six months. The remaining 27 were told to eat whatever they desired for breakfast. Cholesterol scores were recorded for each participant before and after the study. An independent statistician was on hand for the initial cholesterol screens, but he is unfortunately now missing along with his data.

You have been hired by the company to attend the post-experiment cholesterol screening and you are given the data. You are charged with evaluating the company's claim that continuous oatmeal consumption will lower cholesterol scores. While on site you notice that the treatment group seems a bit more spry and healthy. Before each participant leaves you have the good sense to ask each one for their age.

The data from the cholesterol experiment are available in the cholesterol.xls file on the ESM 206B website. The following table defines the variables.

<u>Variable</u>	<u>Definition</u>
Treatment	= Y if the person was told to eat oatmeal each day, otherwise = N
Cholesterol	The Cholesterol score of each participant recorded after the experiment
Age	The age of each participant

Input the data into JMP for analysis.

Puritan Oatmeal claims that selection into the study, and into treatment and control groups, was random. You begin your analysis by assuming that the company's claim is true.

1. What simple test can you use to evaluate the average treatment effect? Briefly defend your choice? (hint: think ESM 206A – is this a one-sample or two-sample test)?

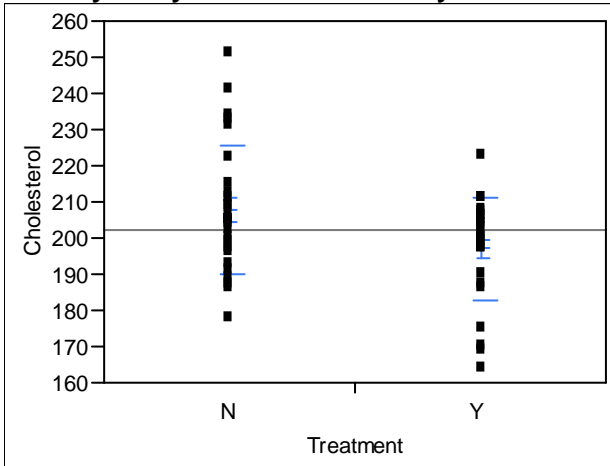
You can use a two-sample t test of means to determine whether or not there is a difference in the mean cholesterol for each group.

2. What do you conclude from your simple test? Does eating oatmeal every day lower cholesterol? Briefly comment on the internal and external validity of your findings? That is, can the findings be applied to males or other age groups or to other communities?

In JMP I conduct a t test of means under Analyze -> Fit Y by X. The results show that mean cholesterol for the treated group is 10.8 lower than for the control group. This difference is statistically significant at $P = 0.016$ for a two-tailed hypothesis test (see JMP output below). Hence, we would reject the null hypothesis that the treatment has no effect on cholesterol.

The finding is an average treatment effect for the population of females, age 21-40, in Cheesetown, Wisconsin. Although the findings may be valid for other communities and age groups, it is risky practice to extend the findings outside of the setting of the specific study.

Oneway Analysis of Cholesterol By Treatment

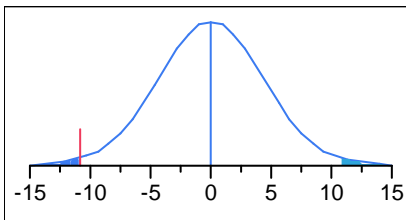


Means and Std Deviations

Level	Number	Mean	Std Dev	Std Err Mean	Lower 95%	Upper 95%
N	27	207.815	17.7938	3.4244	200.78	214.85
Y	27	196.963	13.9408	2.6829	191.45	202.48

Assuming unequal variances

Difference	-10.852	t Ratio	-2.49454
Std Err Dif	4.350	DF	49.18353
Upper CL Dif	-2.111	Prob > t	0.0160
Lower CL Dif	-19.593	Prob > t	0.9920
Confidence	0.95	Prob < t	0.0080

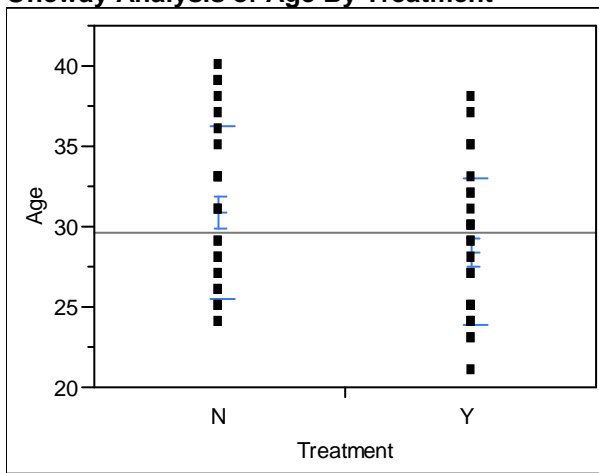


Puritan Oatmeal is satisfied with your findings and wants you to write up the results without further analysis. Although you are tempted by the large paycheck, you decide to first investigate whether the assignment into treatment and control groups is systematically related to age.

3. Perform a simple test to see if the average age is different between the treatment and control group.

In JMP I conduct a two-sample t test of means under Analyze -> Fit Y by X. The results show that mean age for the treated group is 2.4 years lower than the control group. This difference in means is statistically significant at $P = 0.078$ for a two-tailed hypothesis test (see JMP output below).

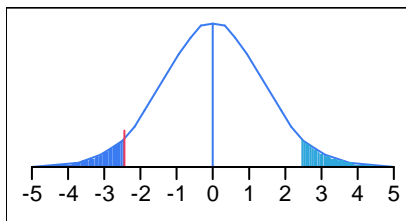
Oneway Analysis of Age By Treatment



Means and Std Deviations

Level	Number	Mean	Std Dev	Std Err Mean	Lower 95%	Upper 95%
N	27	30.8519	5.36874	1.0332	28.728	32.976
Y	27	28.4074	4.59313	0.8839	26.590	30.224

Difference	-2.4444	t Ratio	-1.79773
Std Err Dif	1.3597	DF	50.78344
Upper CL Dif	0.2856	Prob > t	0.0782
Lower CL Dif	-5.1745	Prob > t	0.9609
Confidence	0.95	Prob < t	0.0391



4. What do you conclude? Does this make you skeptical of the company's random selection claim?

Although the difference in means is not significant at $\alpha=0.05$, it is very close (because $P = 0.078$). So this provides some evidence that there was selection bias and that younger females tended to be selected into the treatment group.

Even if the random selection claim is true, you want to use the data you have on age to determine (a) if a significant average treatment effect still exists when we control for age, and (b) whether the treatment effect varies with age.

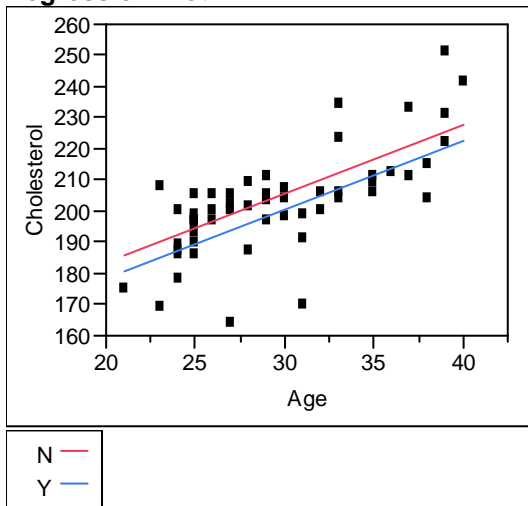
5. Which statistical tool described in lecture can you use to evaluate (a) and (b)? (Hint: we have both a categorical and continuous variable).

ANCOVA is an appropriate tool when we have continuous and categorical variables. (Regression is another option and that approach and interpretation is slightly different).

6. What do you conclude with respect to (a)? That is, is the average effect of treatment on cholesterol robust to controlling for age?

In JMP, I use Analyze->Fit Model -> Y is cholesterol and X is Treatment and Age. Here we find only weak evidence that treatment is associated with lower cholesterol scores once we control for age. The P value is 0.102, which is larger than our usual rejection criteria of 0.05.

**Response Cholesterol
Whole Model
Regression Plot**



Actual by Predicted Plot

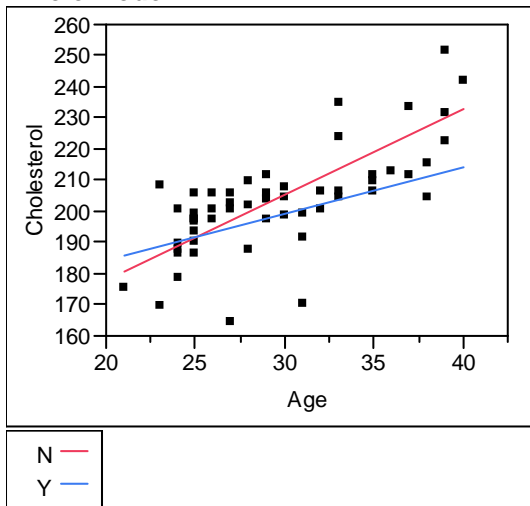
Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	136.85603	9.721479	14.08	<.0001
Treatment[N]	2.7226957	1.635543	1.66	0.1021
Age	2.2117338	0.323699	6.83	<.0001

7. What do you conclude with respect to (b)?

*In JMP, I use Analyze->Fit Model -> Y is cholesterol and X is Treatment, Age, and Age*Treatment. Here we find some evidence that the treatment mitigates the increase in cholesterol that is associated with increases in age. The P value on the interaction term is 0.063, which is only a bit larger than our usual rejection criteria of 0.05.*

**Response Cholesterol
Whole Model**



Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	138.89935	9.542321	14.56	<.0001
Treatment[N]	2.8376083	1.596328	1.78	0.0816
Age	2.1177144	0.319562	6.63	<.0001
Treatment[N]*(Age-29.6296)	0.6074511	0.319562	1.90	0.0631

8. Given your analysis here, briefly describe the strongest conclusion you can make that is favorable to Puritan Oatmeal. Also briefly describe the strongest conclusion you can make that is unfavorable to Puritan Oatmeal.

Most favorable conclusion for Puritan Oatmeal, Inc.

There is some evidence suggesting that the treated group had lower average cholesterol even when we control for differences in age. Although this evidence isn't statistically significant at conventional levels, it is quite close at 0.10. Moreover, we find that eating oatmeal for breakfast every day mediates the positive relationship between age and cholesterol levels. The mitigation effect is not statistically significant at 0.05, but it is at 0.06.

Least favorable conclusion for Puritan Oatmeal, Inc.

The difference in mean ages between the two groups suggests that selection was not random, and this implies that pre-experiment cholesterol levels were probably different across the two groups. In light of this, we cannot assign differences in post-experiment cholesterol scores to treatment.

9. Now suppose you track down the independent statistician. He tells you that the mean pre-experiment cholesterol for the ‘treatment’ group was 210.1 and the mean cholesterol for the ‘control’ group was 210.6. The difference in pre-experiment means was not at all close to being statistically significant. How does this change your most and least favorable conclusions?

Most favorable conclusion for Puritan Oatmeal, Inc.

There was not a significant difference in pre-experiment cholesterol means between the two groups, but the post-treatment mean cholesterol for the treated group is lower. This is strong evidence that treatment improved average scores across the entire age range. Moreover, we find that eating oatmeal for breakfast every day is relatively more beneficial as age increases.

Least favorable conclusion for Puritan Oatmeal, Inc.

Although we can’t deny the evidence that scores improved more for the treated group, we also can’t make general conclusions about the effect of treatment because the assignment into groups was not random. At best we can say that eating oatmeal is beneficial for this particular treatment group of 27 compared to the particular control group of 27. (That is, we cannot rule out the possibility that the treatment group was comprised of females whose cholesterol scores will be most responsive to eating oatmeal, which is why the results can’t be generalized to other populations).