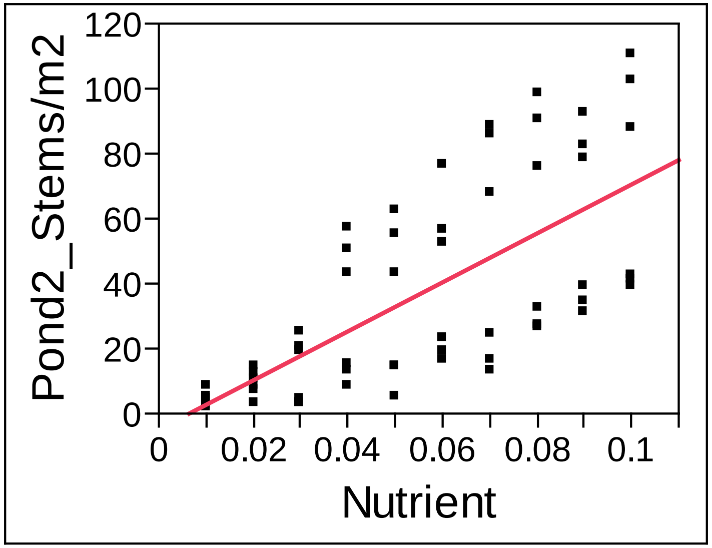
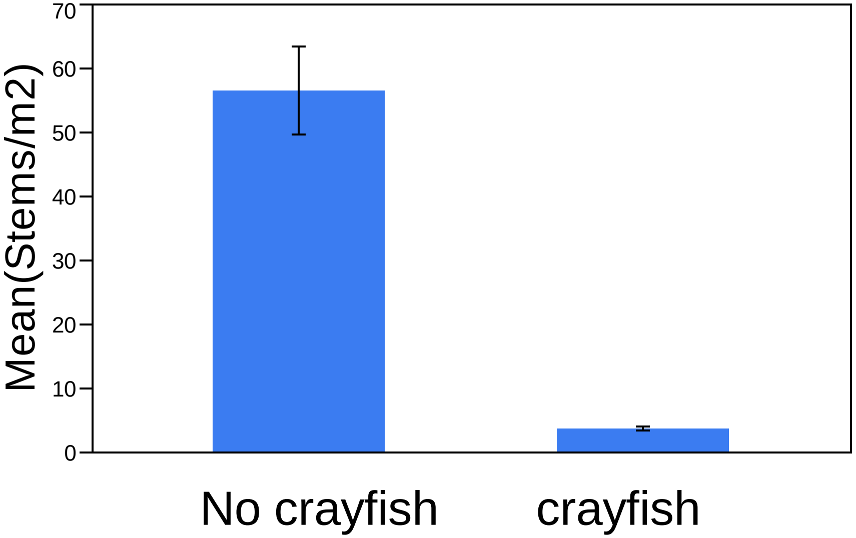
Analysis of Covariance (ANCOVA)

Hybrid of ANOVA and regression





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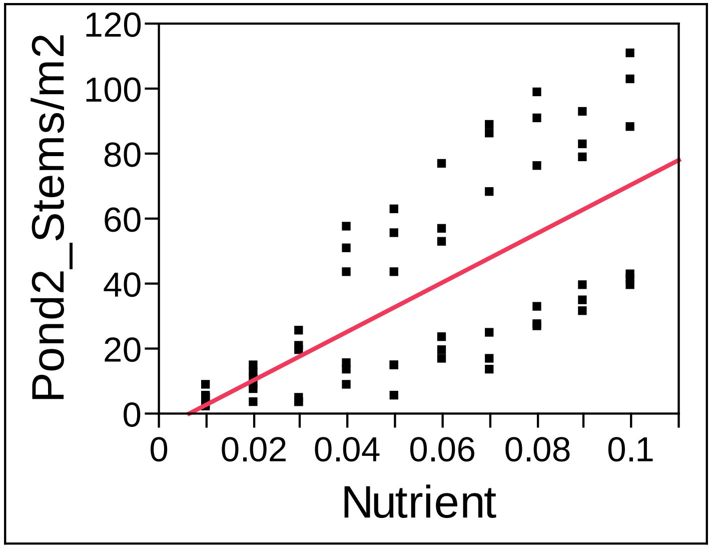
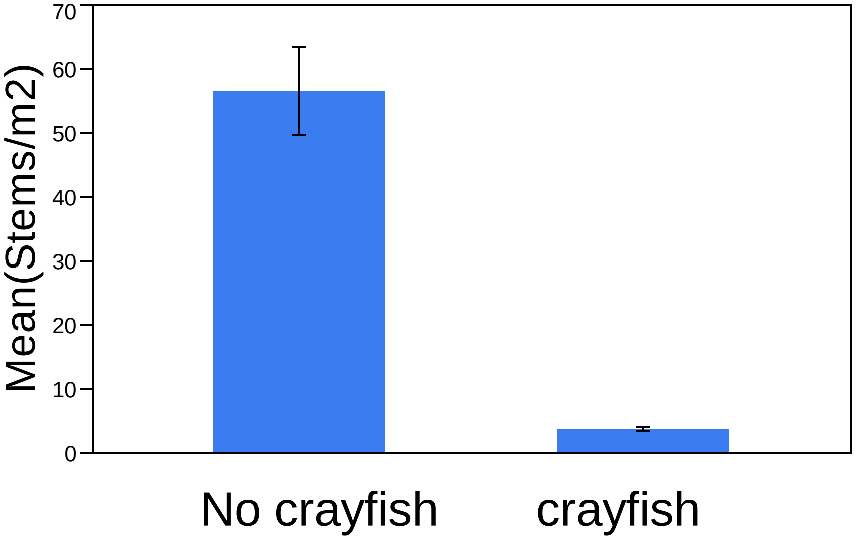


You have both categorical and continuous X variables



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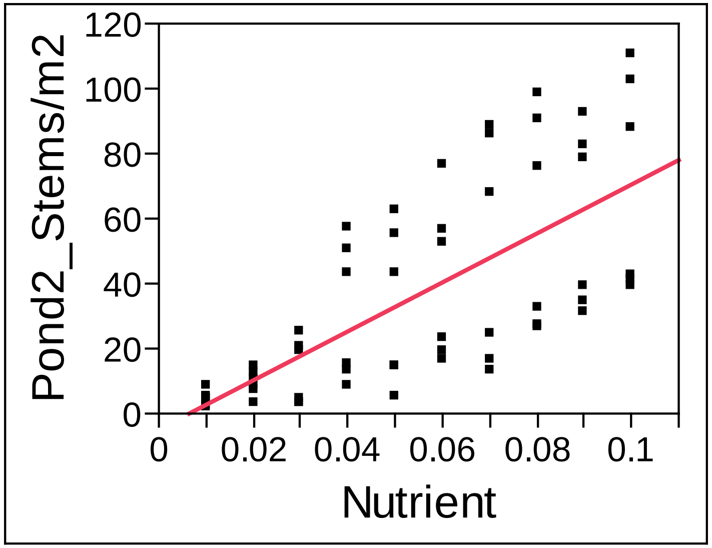

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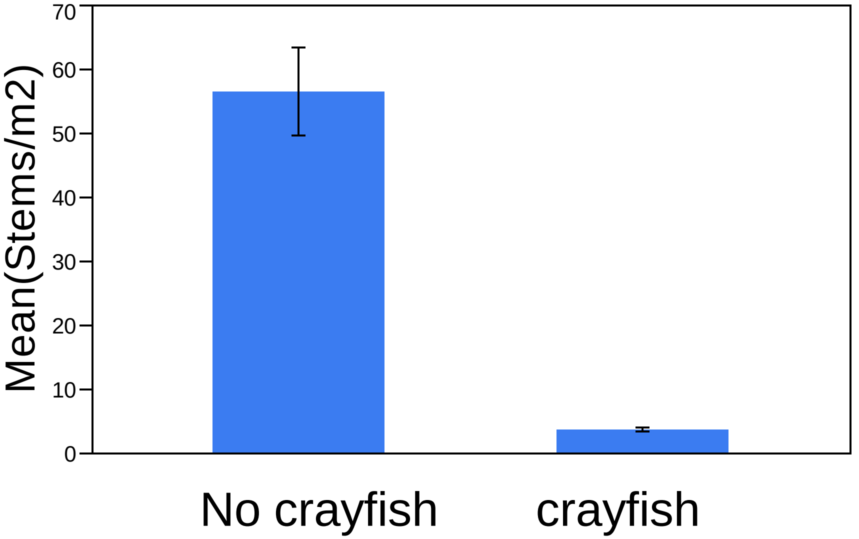
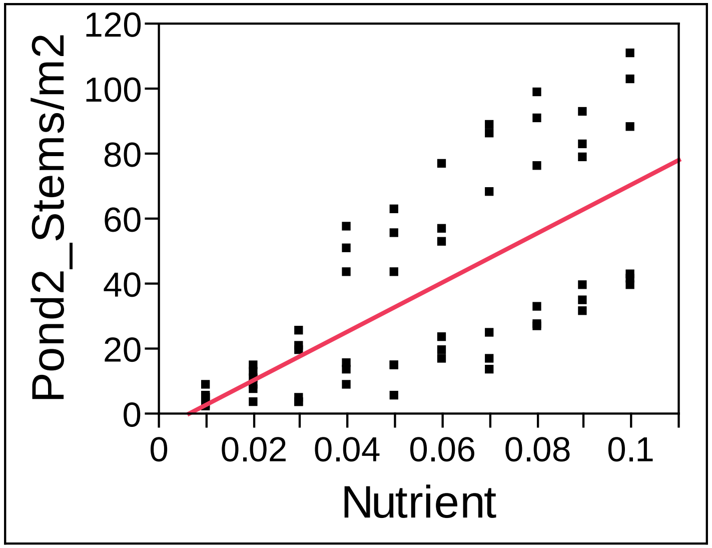
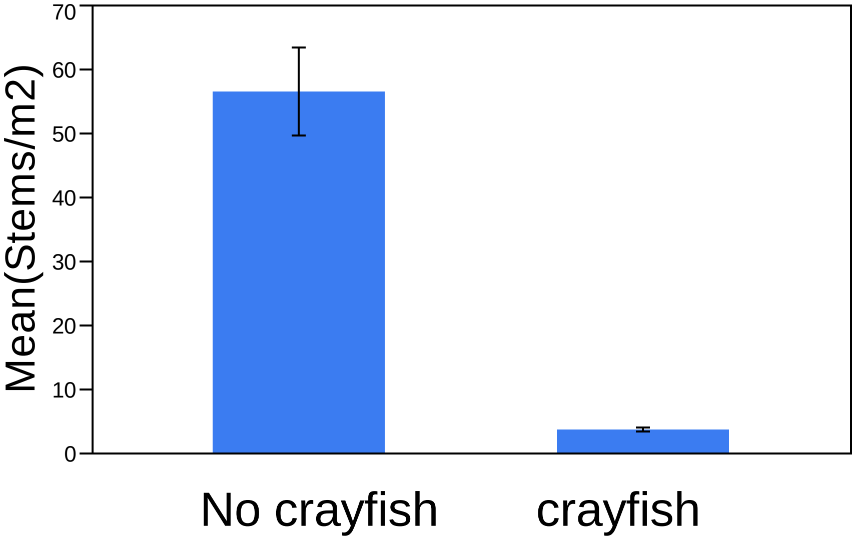


ANOVA design, with an additional continuous variable (covariate)

Analysis of Covariance (ANCOVA)

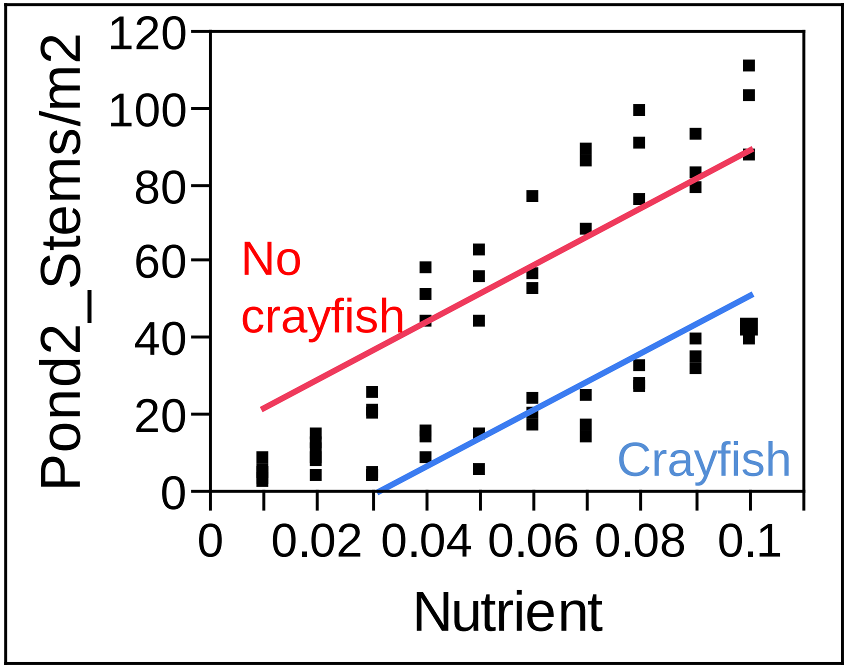


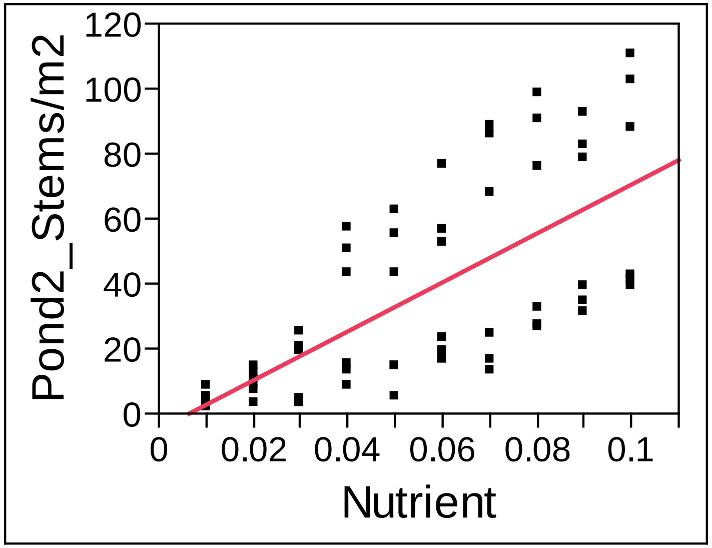


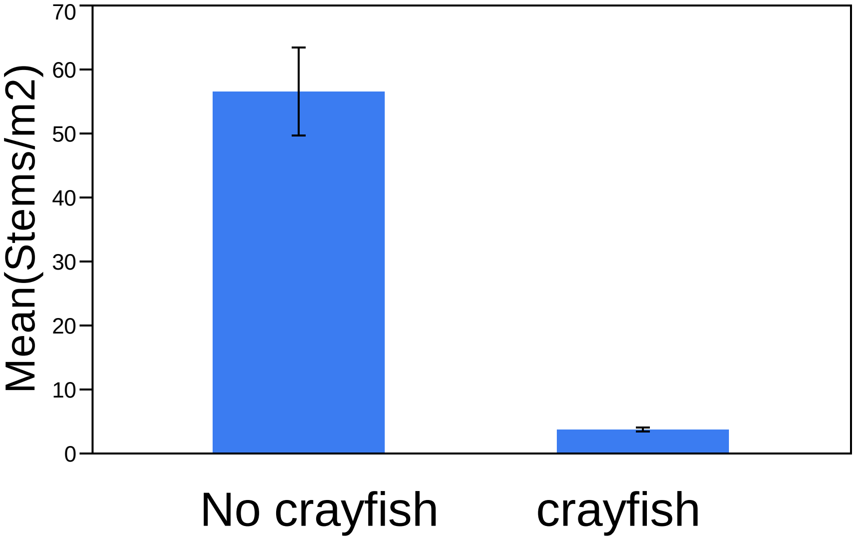
ANOVA design, with an additional continuous variable (covariate)

Photo: E. Engbretson, U.S. Fish and Wildlife Service

Analysis of Covariance (ANCOVA)







ANOVA design, with an additional continuous variable (covariate)

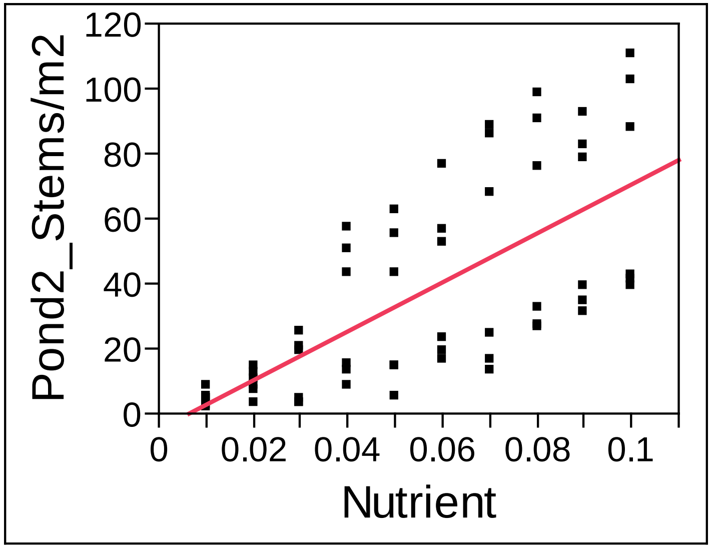
fake_crayfish.jmp

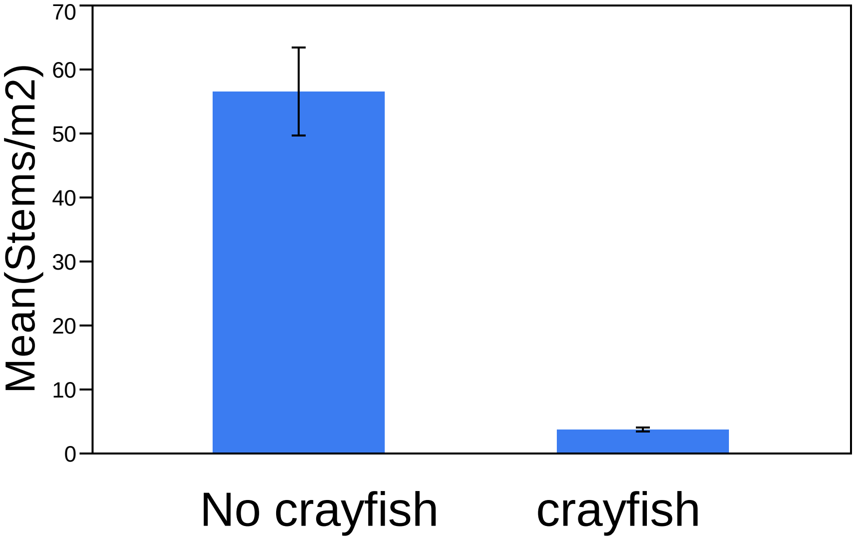
The modification of ANOVA that allows ANCOVA

ANOVA

$Y_{ij} = \mu + A_{ij} + \varepsilon_{ij}$

Observations Y are a combo of some mean level μ + the positive or negative average effect A of each level i plus error ε





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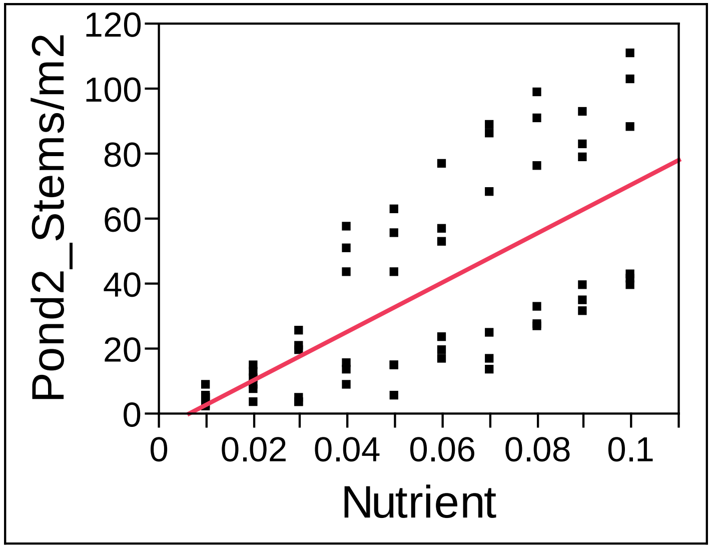

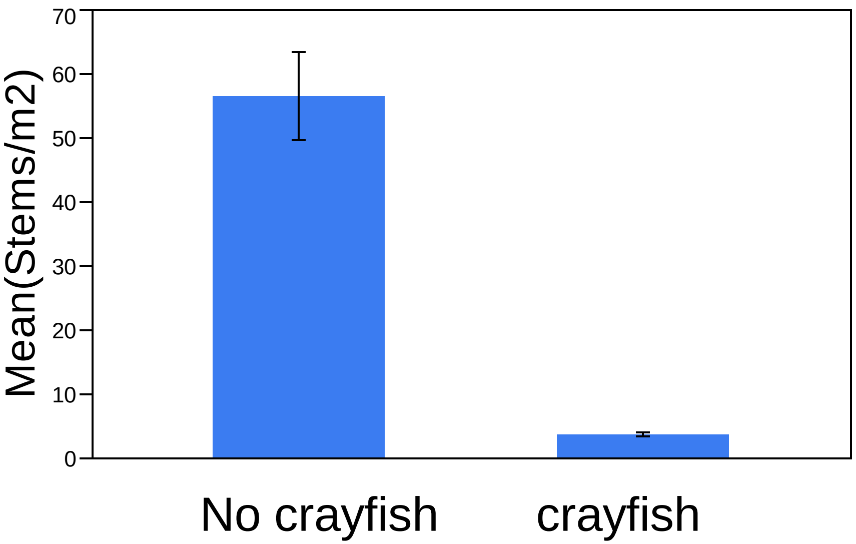

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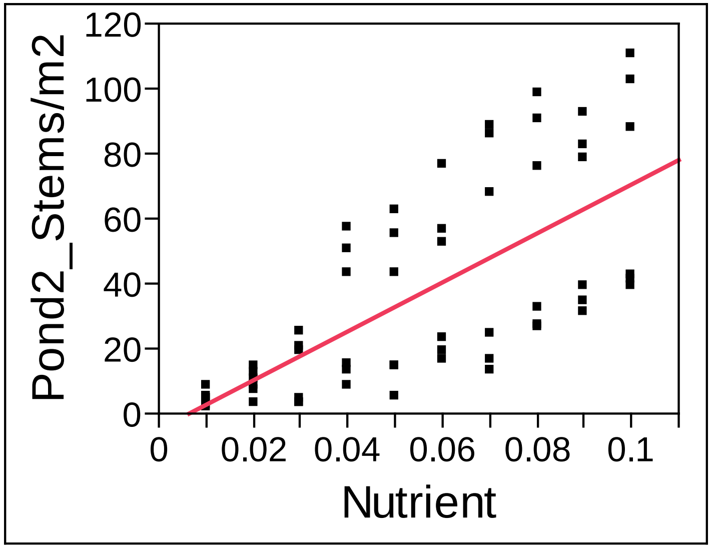

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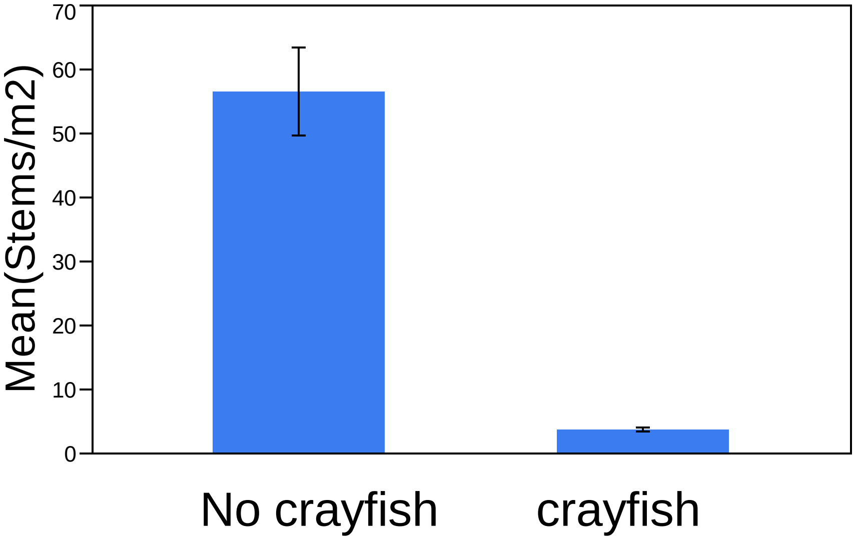
So… correlate Y with the continuous variable first, and then use ANOVA to analyze residuals



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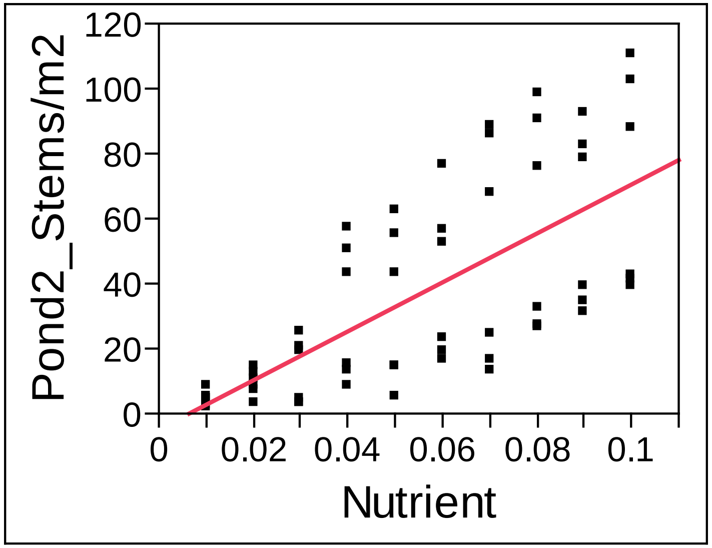

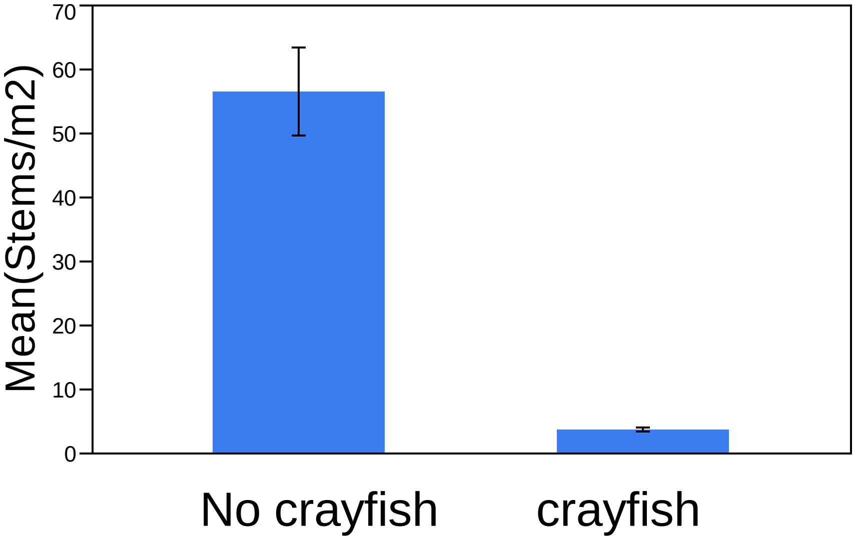

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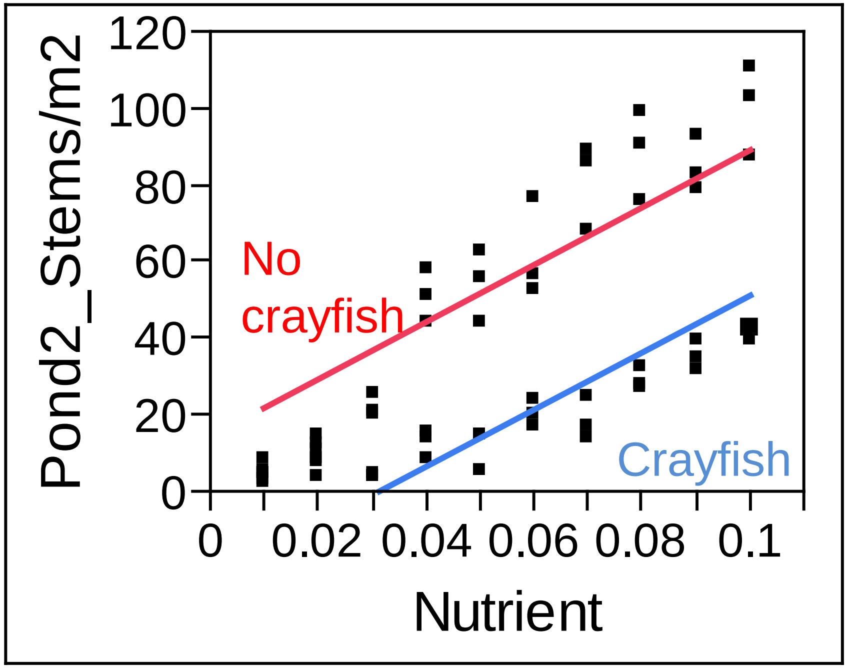

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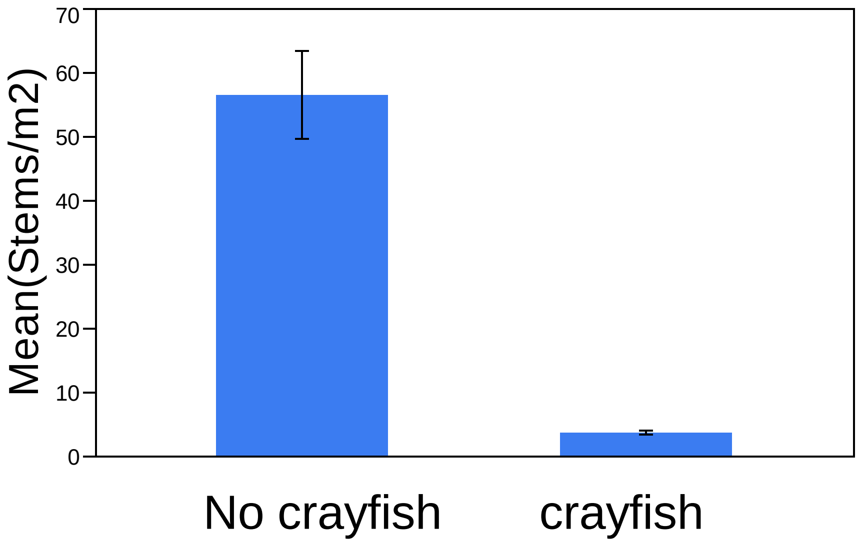

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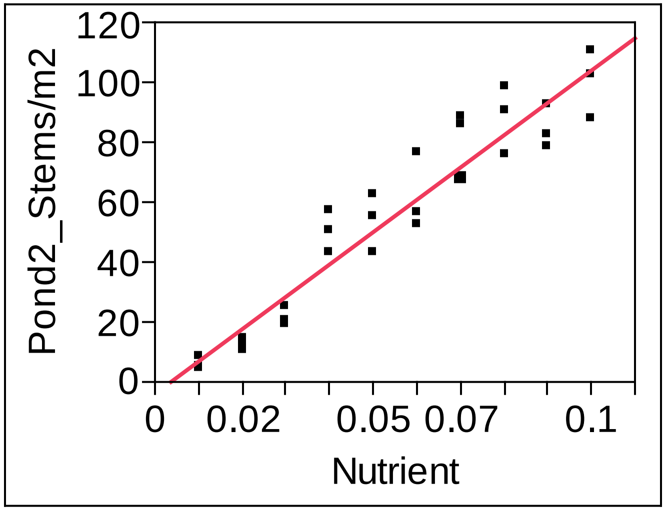

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If the treatment effect (A) is non-significant, the equation may reduce to a linear regression

fake_crayfish.jmp

ANCOVA

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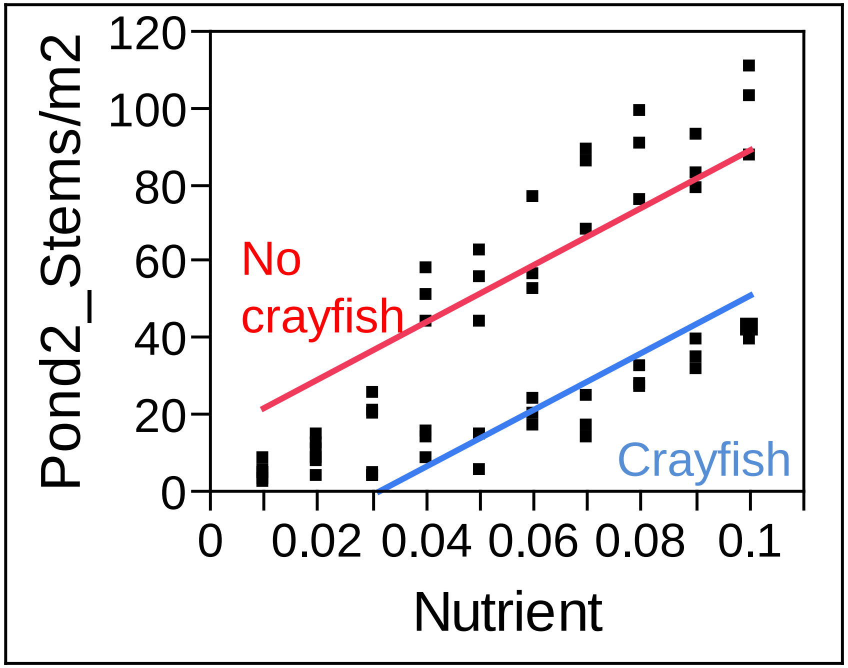
Generally, the algorithm first tests to see whether slopes differ between/among treatments

If not, a common slope is used – the simplest case



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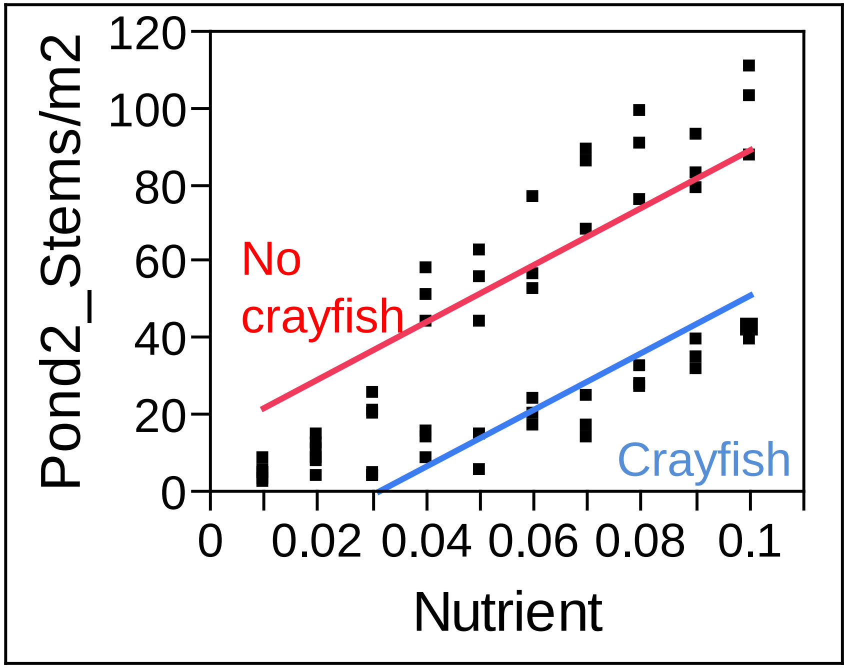
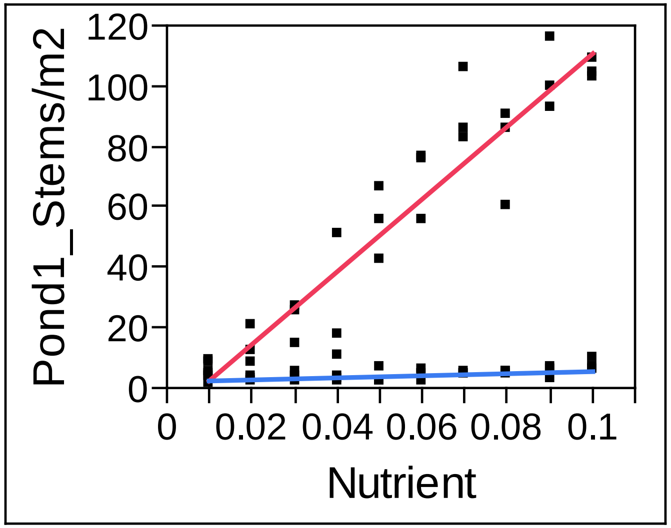
Interaction term is significant when slopes differ significantly





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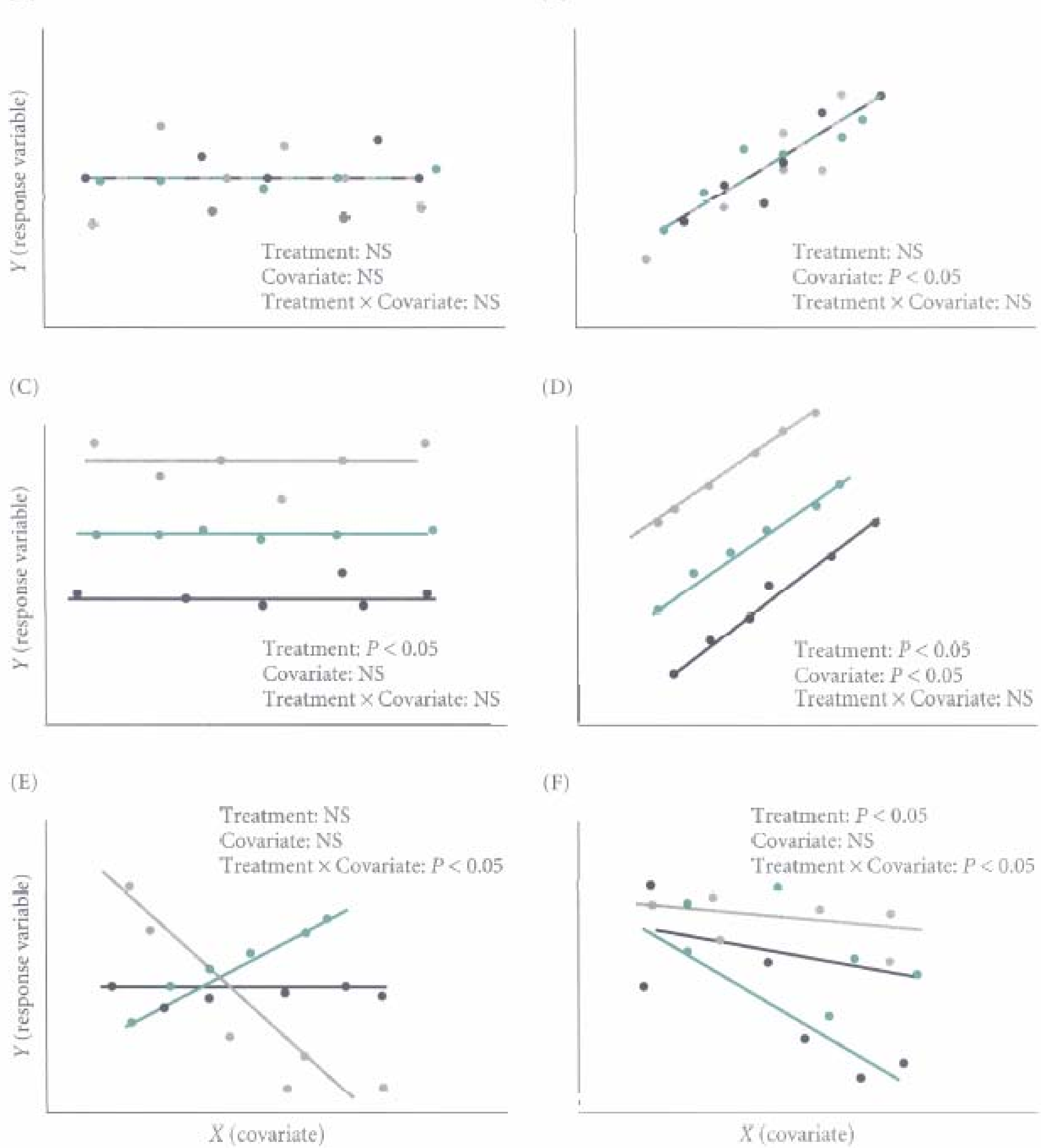
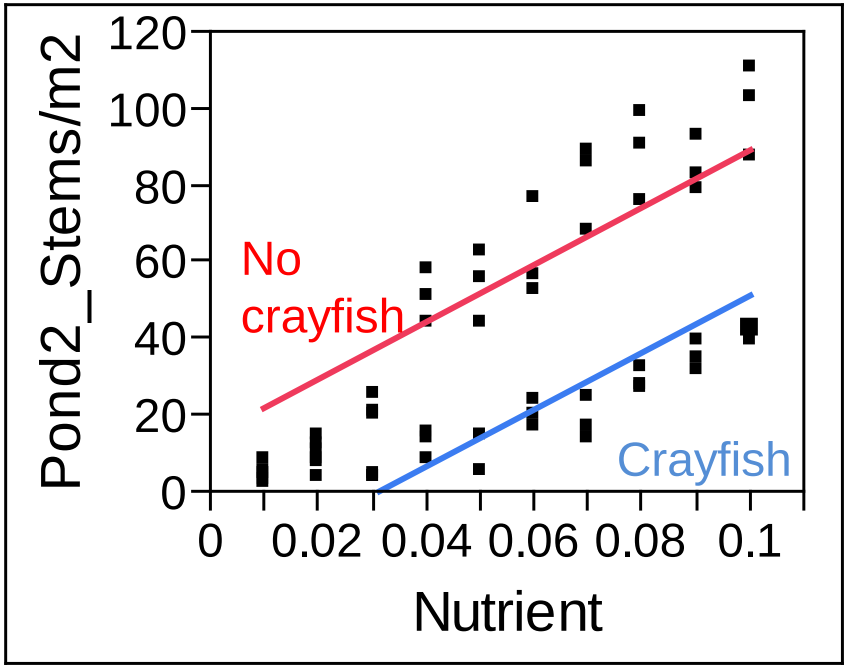


Gotelli & Ellison Fig. 10.4

What are important assumptions in ANCOVA?

Normality/homoscedasticity

Independent samples



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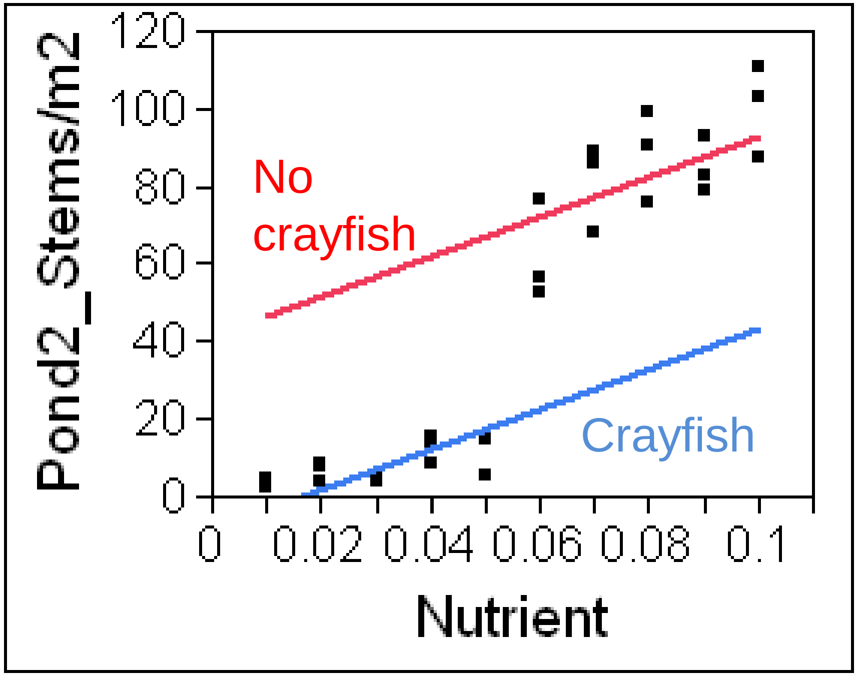

Careful that effects are not confounded!

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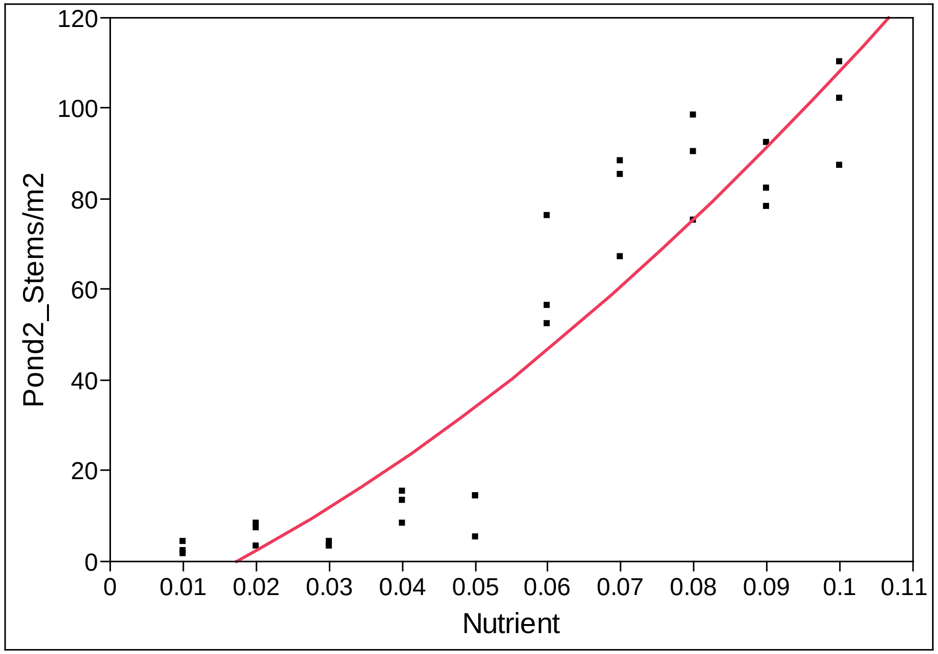

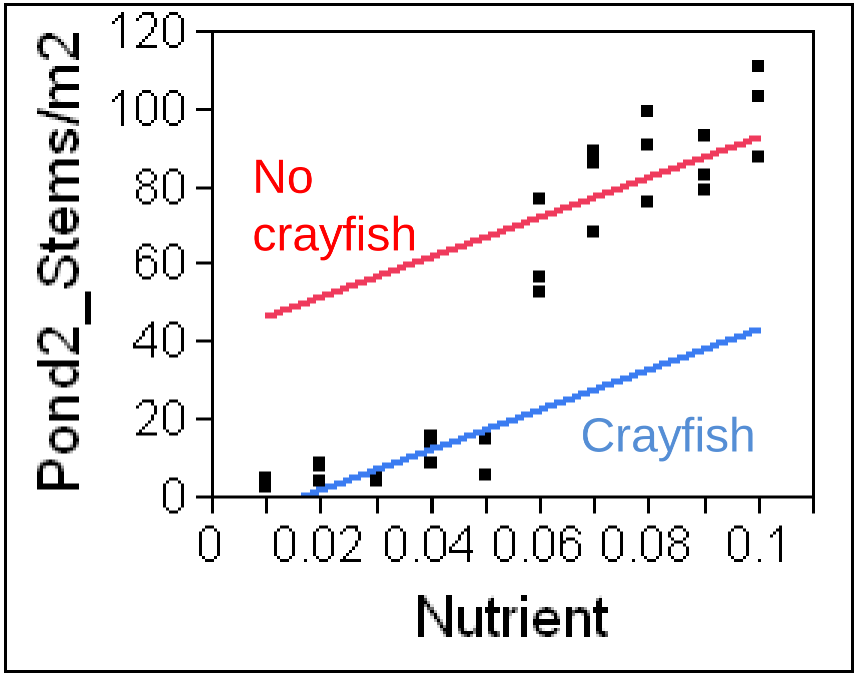

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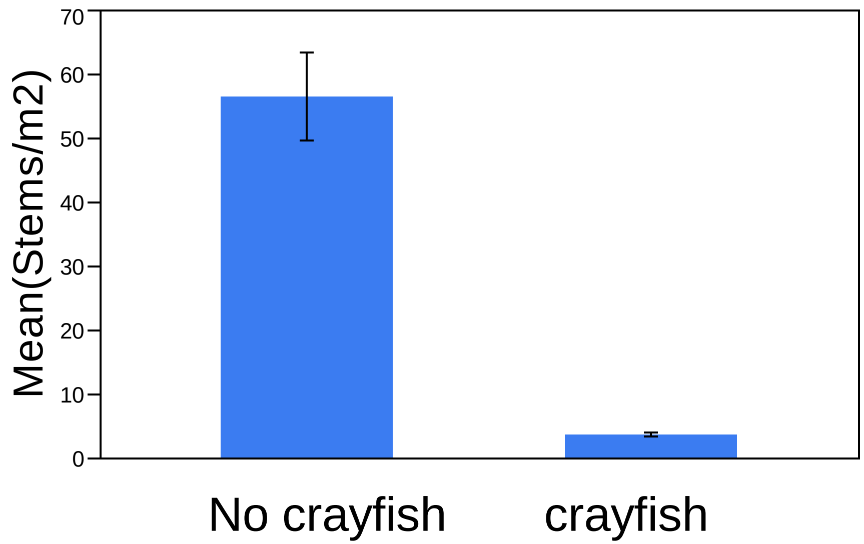

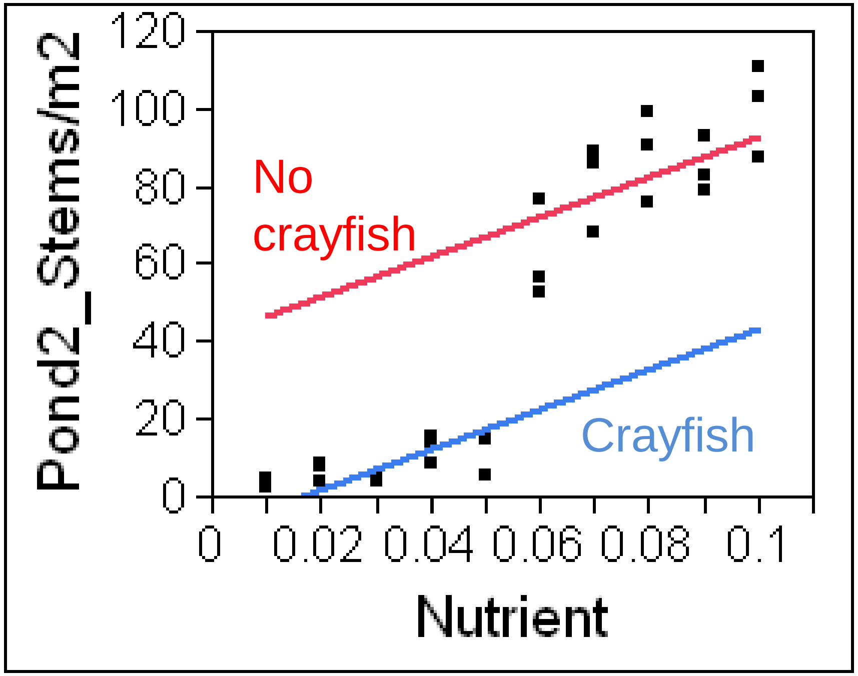

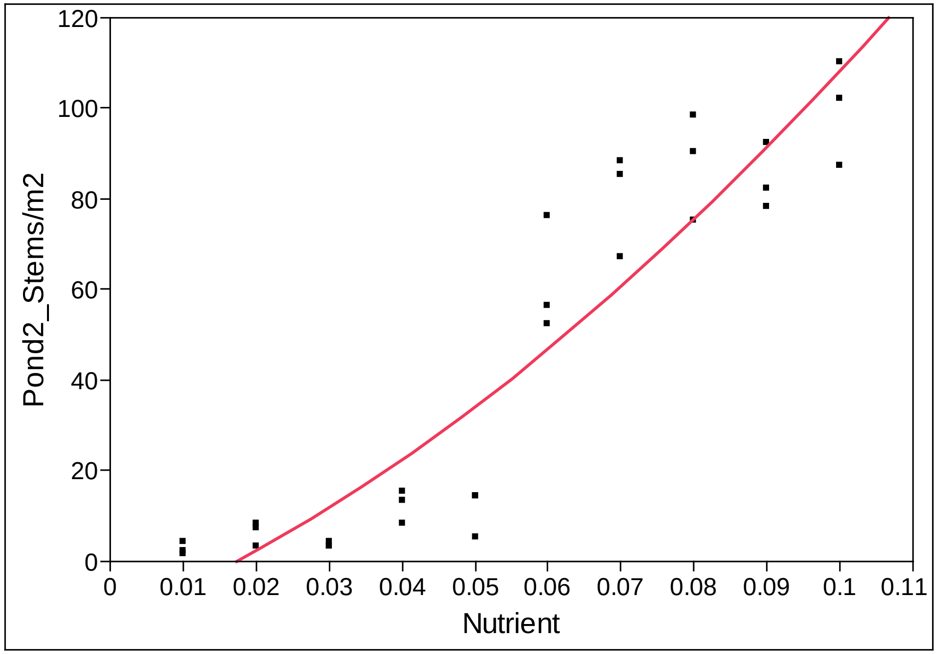

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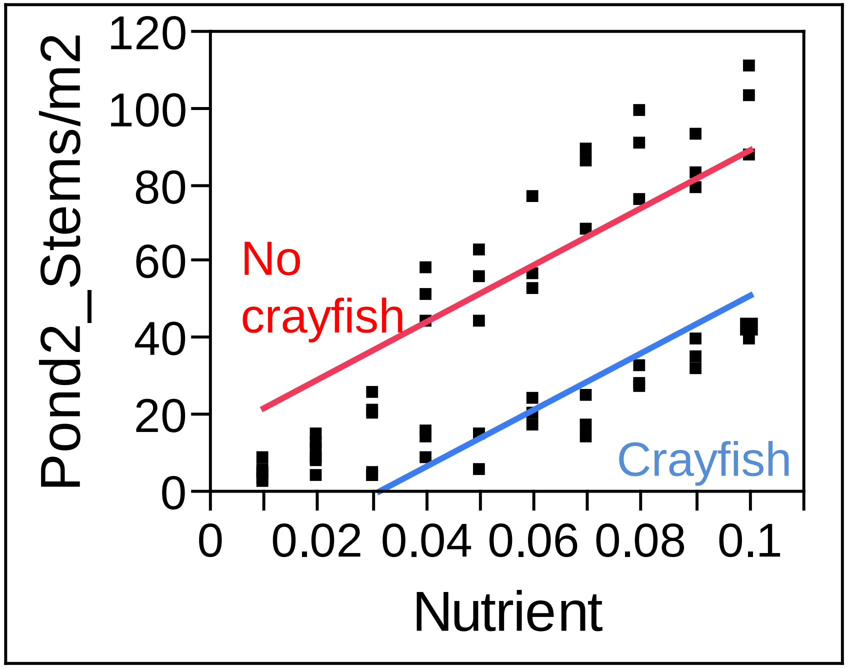
$Y_{ij} = \mu + A_i + ß_i(X_{ij} - X_i) + \varepsilon_{ij}$

May be used in a range of situations, e.g.,

-to explain "extra" variation in experiment where the ANOVA design is of greatest interest



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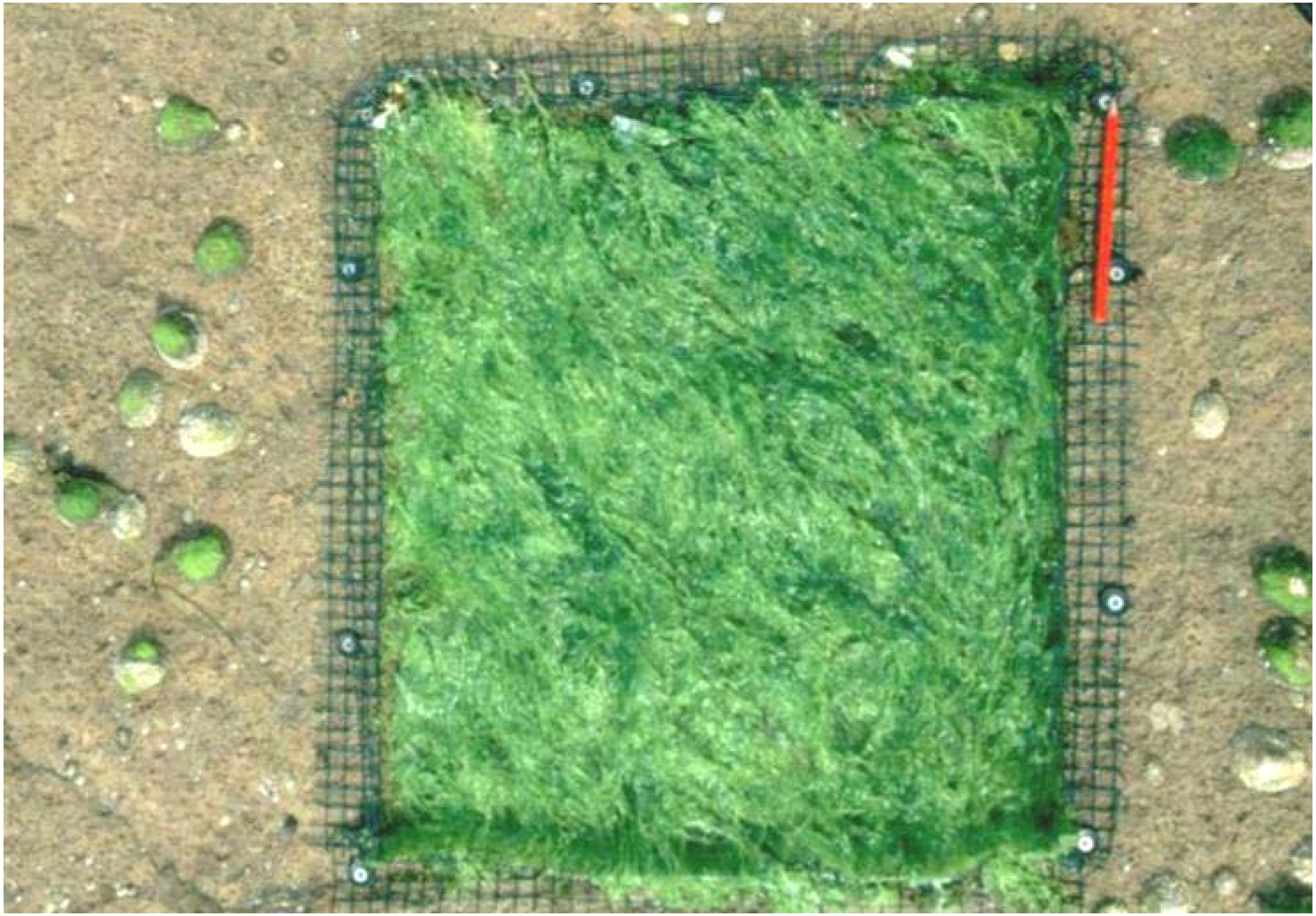
e.g.,
Presence/Absence of grazer in field exclosures
+ naturally varying temperature, slope, or nutrients….



http://www.field-studies.org.uk/limpet_grazing.htm

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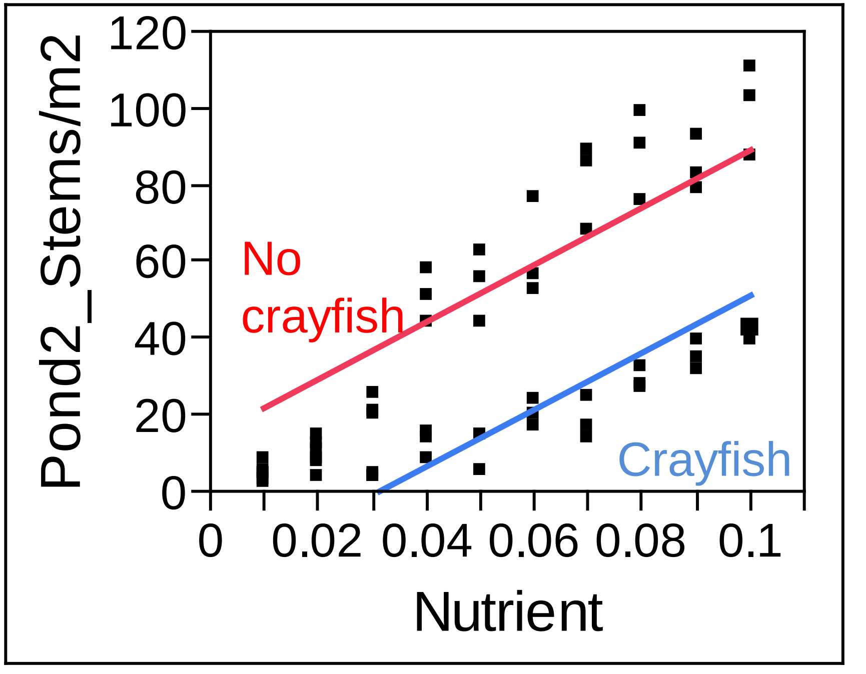

-experimentally, where continuous and categorical variables are both manipulated

See: Gotelli & Ellison P. 197

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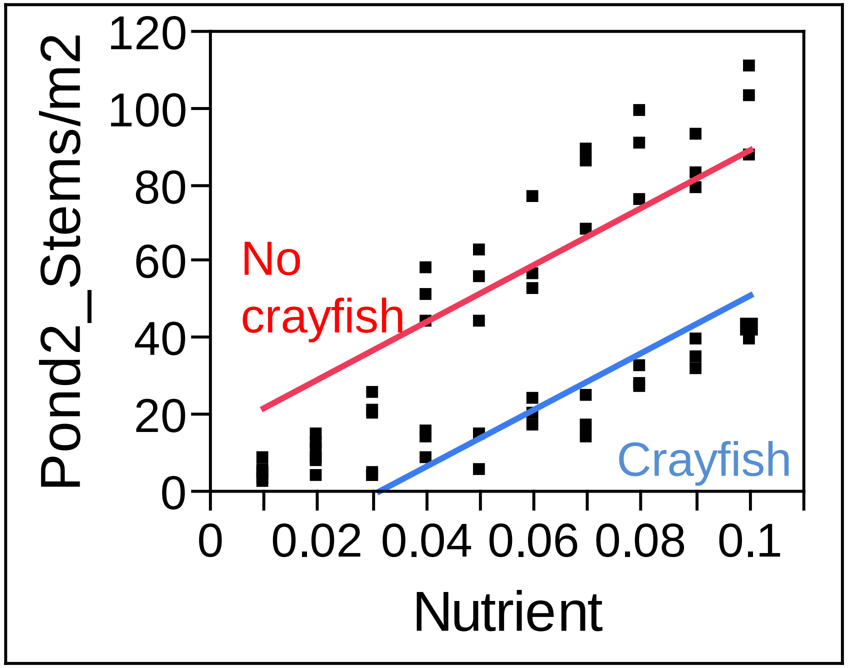

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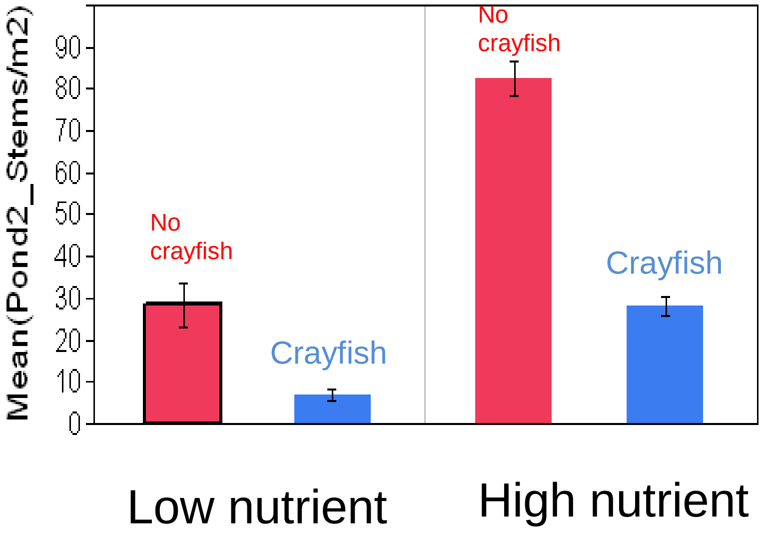

If you try to "shoehorn" a continuous variable into discrete categories for a 2-way ANOVA, you will need to treat that factor as **random** rather than **fixed**

See: Gotelli & Ellison P. 199, Footnote

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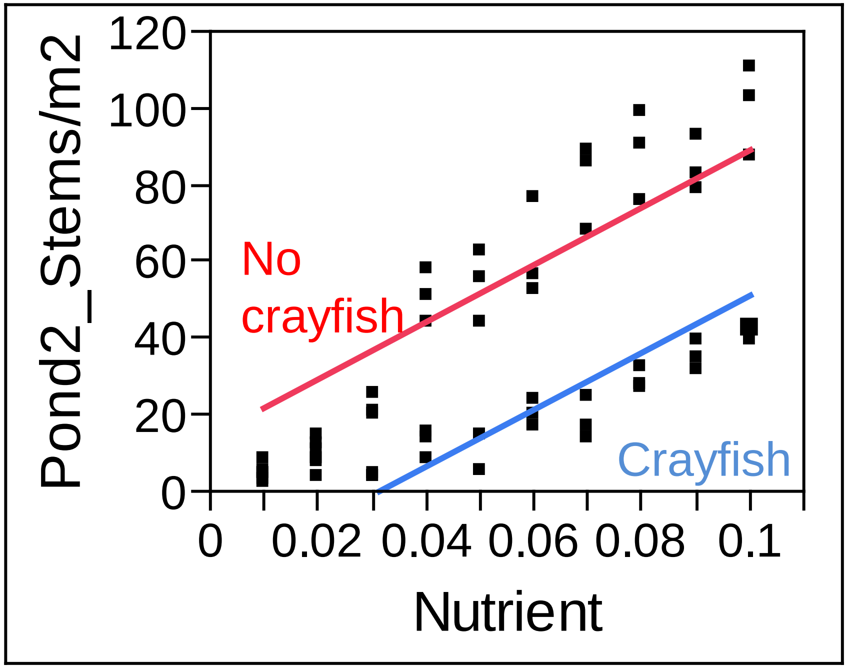

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Random – the treatment levels represent a random sample of all possible levels – inferences valid across other levels

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Fixed – the levels included are the ONLY levels of interest

See: Gotelli & Ellison P. 199, Footnote 10

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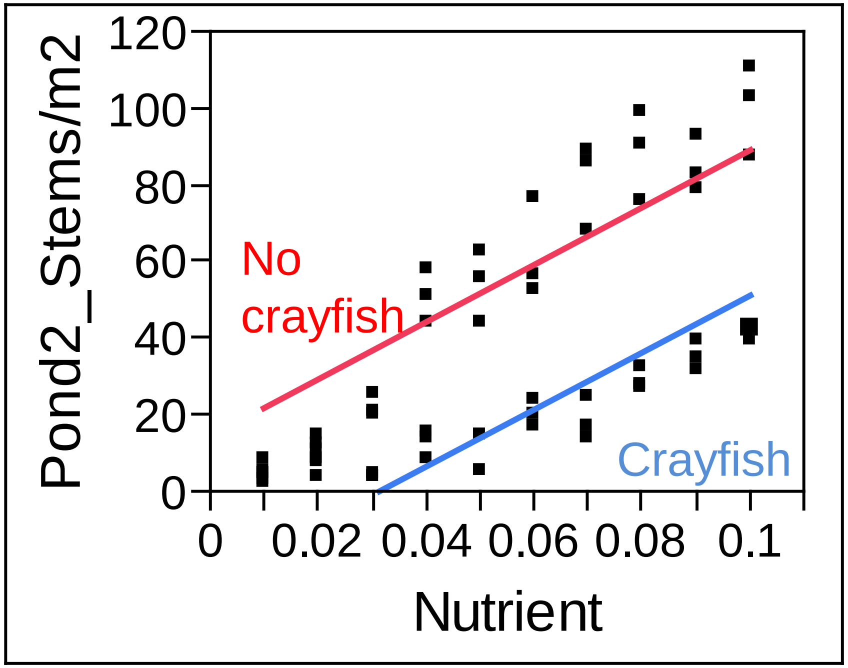

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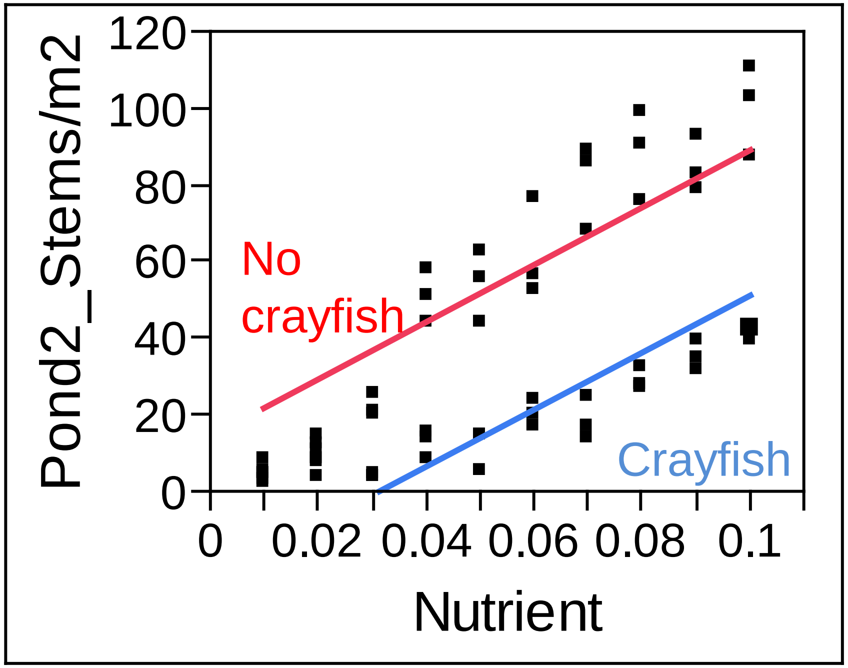

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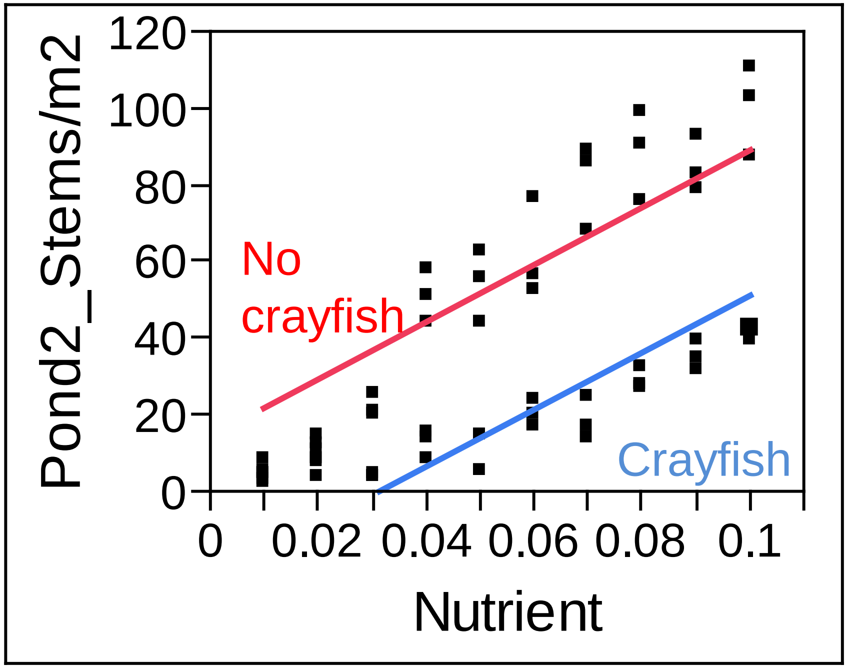
Who cares?! Calculations differ, significance values change dramatically

Why is it important to identify fixed vs. random factors?

Constructing F-values:

Fixed effect:

$$\frac{\text{Mean square}_{factor}}{\text{Mean square error}}$$



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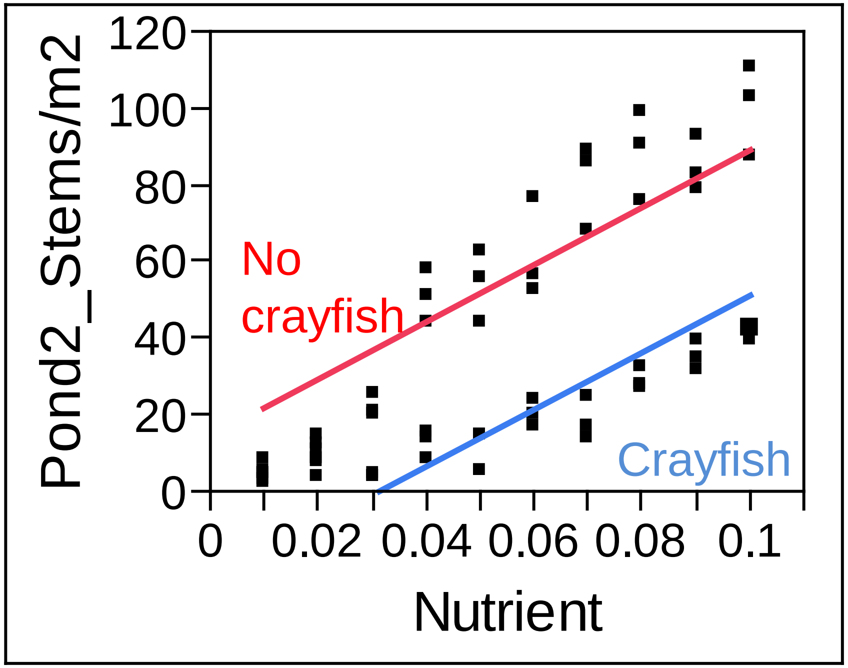

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Neter et al. p. 982
Gotelli & Ellison p. 319

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Constructing F-values:

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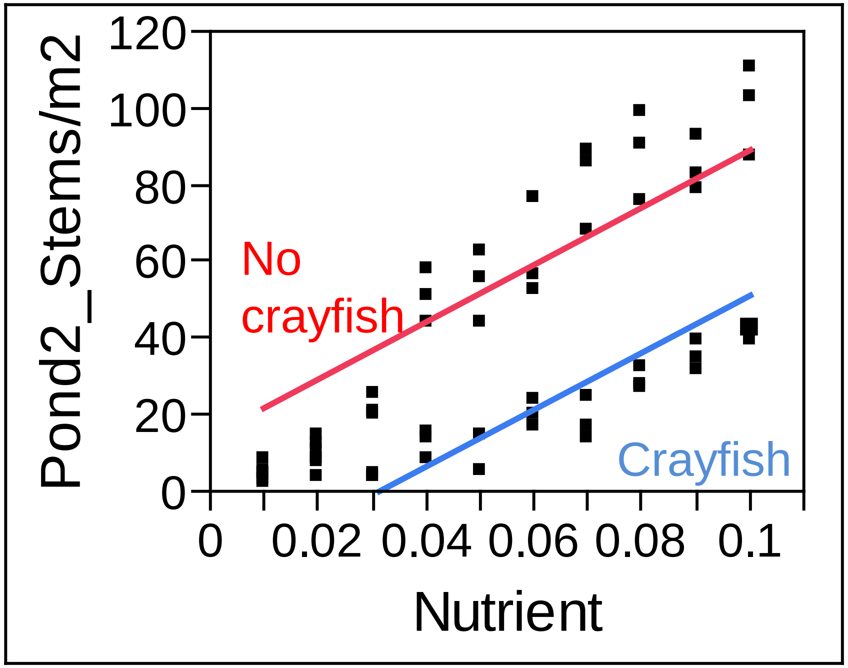
$$\frac{\text{Mean square}_{\text{factor}}}{\text{Mean square error}}$$

Random effect:

$$\frac{\text{Mean square}_{\text{factor}}}{\text{Mean square}_{\text{interaction term}}}$$

Mean $\text{square}_{\text{interaction term}}$ is strongly affected by # levels and not by *n* – usually a bigger number, so F-value smaller (less likely to be significant)



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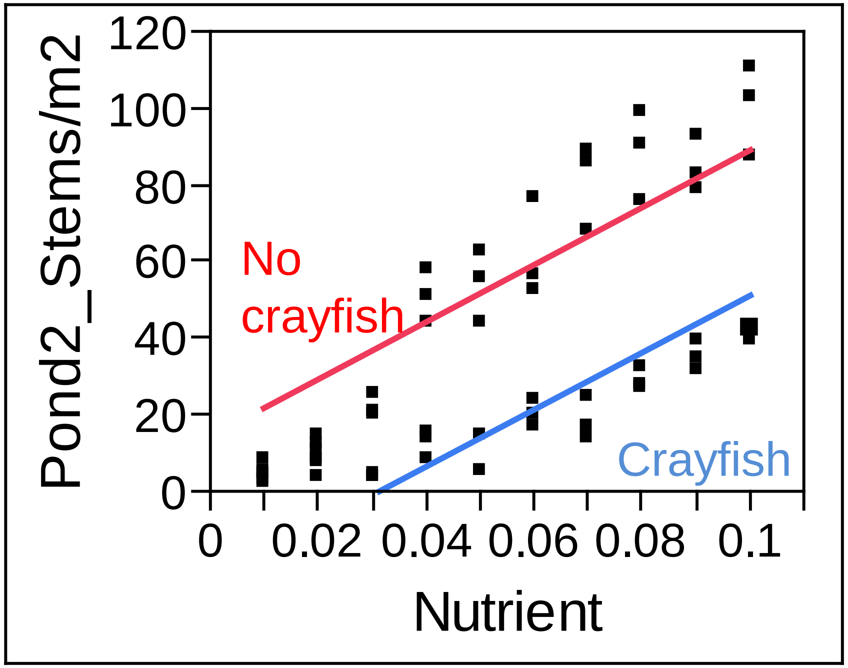
Moral:

Do regression when regression makes sense

Do ANOVA when ANOVA makes sense

Do ANCOVA when ANCOVA makes sense



Moral:

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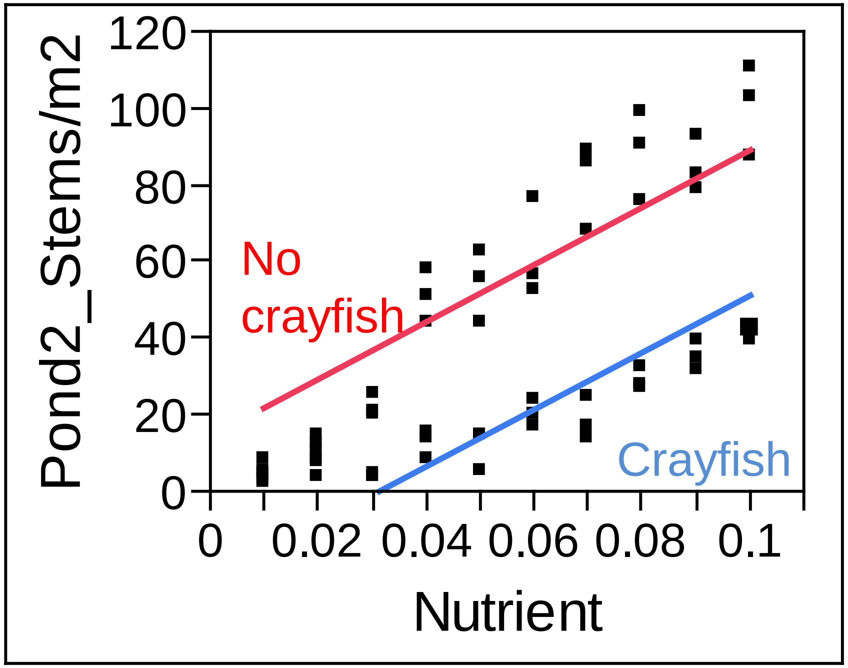
Think about whether effects are random or fixed BEFORE you do your study – it may change your sampling approach



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Fixed – **the levels included are the ONLY levels of interest**

Who cares?! **Calculations differ, significance values change dramatically**

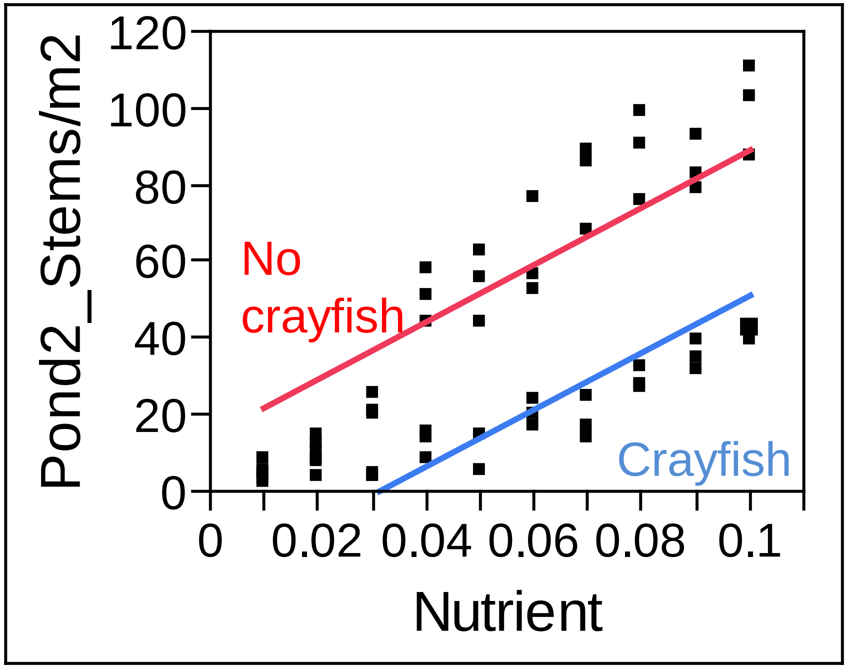
See: Gotelli & Ellison P. 319

More resources

ANCOVA, fixed/random effects:

Gotelli & Ellison. A Primer of Ecological Statistics. Sinauer.

Neter et al. Applied Linear Statistical Models. McGraw Hill.



Fixed/random effects in designing experiments with regression:
Cottingham et al (2005) Knowing when to draw the line: designing more informative ecological experiments. Frontiers in Ecology and the Environment. 3(3): 145–152
http://microbes.kbs.msu.edu/Publications/Cottingham%20et%20al.%202005.pdf

More about what fixed and random effects are, especially for ecologists:
Bennington & Thayne. 1994. Use and Misuse of Mixed Model Analysis of Variance in Ecological Studies. Ecology 75(3): 717-722.
http://www.jstor.org/stable/1941729