ESM 242: Assignment #3
Due in class Monday, May 25, 2009

1 A Stylized Climate Change Model

Let $\tau(t)$ be the temperature at time t relative to the temperature at time 0 (today). Suppose $\tau(t)$ evolves as follows:

$$\tau(t) = \frac{T}{S}t \quad \text{for} \quad t < S \tag{1}$$

$$= T \quad \text{for} \quad t \geq S \tag{2}$$

where T is the temperature at time S. Under this specification, temperature begins to increase immediately, it increases linearly so that it reaches temperature T at time S, and after time S it remains at temperature T.

Temperature increases adversely affect economic activity, captured here by consumption. The fraction of consumption retained, as a function of temperature is:

$$L(t) = exp(-\beta\tau(t)^2) \tag{3}$$

Consumption grows at constant rate, g, but is proportionally discounted by the above expression. Thus:

$$C(t) = L(t)exp(gt) \tag{4}$$

Utility of consumption is given by the constant relative risk aversion function:

$$U(C) = \frac{C^{1-\eta}}{1-\eta} \tag{5}$$

Remember that utility is a relative measure so it can be negative

Discounting follows the Ramsey rule:

$$r = \delta + \eta g \tag{6}$$

In other words, the discount rate is composed of 2 terms. The first term (δ) is the "pure rate of time preference," which captures the inherent discounting of future utility. The second term (the product of ηg) accounts for the fact that we will be richer in the future (via economic growth, g) *and* that the rich gain less welfare than the poor given an initial baseline. Overall, the discount rate we will use is r, but note that if we change δ, η, and/or g, we will have to change r accordingly.

Consider a base case with the following parameters: $\delta = .01$, $\eta = 3$, $g = .015$, $S = 100$, $\beta = .0017$

2 Exploring the functions

1. What is a reasonable range for T under business as usual carbon emissions? (Note: you may want to consult the latest IPCC report or other publications)

2. What is the *largest* value of T you can defend with a citation? What is the citation?

3. Plot $\tau(t)$ for several values of T spanning the range that you identified above.

4. Plot consumption $C(t)$ without climate change and with climate change for several values of T.

5. Plot the utility function. What is the elasticity of utility with respect to consumption? What is the marginal utility as consumption approaches zero?

3 Setting up the problem

1. Suppose T is known, and it is 4.4. In other words, suppose we knew for sure that under business as usual, the temperature increase would be 4.4 degrees above what it is today. What is the present value of utility with climate change? What is the present value of utility without climate change? What is the loss in utility as a result of climate change (expressed as a percentage loss)?

2. Explore the sensitivity your answer to different values of T, g, S, η, and β. Report the elasticity of utility loss with respect to each of these parameters.

3. Suppose we could completely prevent climate change from occurring today (and thus $T = 0$ instead of 4.4). But to achieve this benefit we have to give up a fraction θ of consumption every year in perpetuity. What is the maximum value of θ we would be willing to endure to completely prevent climate change? Call this value θ^*.

4. How does θ^* depend on T?

5. Is the cost of abatement in this model? Briefly explain.

4 Uncertainty over T

Now suppose there is uncertainty over the parameter T.

1. Pick a statistical distribution that accords with your research on the possible values of T. Plot the density function.

2. What is the *Expected* loss in present value utility from climate change?

3. Now use a fat-tailed distribution with infinite support (e.g. the t-distribution). What is the *Expected* loss in present value utility from climate change?

4. **For Lots of Extra Credit:** Suppose the pdf over T is a Cauchy distribution with shift parameter $\mu = 4.4$ and scale parameter equal to 1. Also suppose it is truncated below by 0 and above by $\bar{T}$. The pdf for this distribution is given by:

$$f(T) = \frac{(1 + (T - \mu)^2)^{-1}}{arctan(\bar{T} - \mu) - arctan(-\mu)} \tag{7}$$

   Compute the value of θ^* for different values of the upper truncation point, $\bar{T}$ in the interval $[0, 90]$.

5 Progress Report

Provide a 1 page report of the current status of your term project. Include a list of what still needs to be done. Submit the progress report separately from this homework.