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# Structural Change and Sustainable Development

*by*

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**Abstract.** In this paper, we show that the commonly observed decline in primary (natural resource using) sector output and employment shares, often termed structural change, can be explained as an endogenous response to the presence of nature's constraint. Structural change takes place even if consumer preferences are homothetic, and technological progress does not discriminate against the primary sector. Under certain conditions, structural change allows an open economy to grow with natural resource sustainability. Sustained and environmentally sustainable economic growth is possible even if the natural resource is exploited under open access. Well-defined property rights are neither necessary, nor sufficient for sustainable growth. We show that there is no unique relationship between natural resource endowment and the rate of economic growth over the long run. Resource-rich economies may grow faster or slower than resource-poor ones.

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## I. Introduction

While some economists remain concerned that continuous growth places unsustainable demands on the natural resources others believe that improving technology makes growth and environmental sustainability compatible. In this paper we revisit this issue and ask three questions. First, given a distortion-free economy that uses a renewable natural resource as factor of production, what makes sustainable growth possible? Second, if the natural resource is characterized by ill-defined property rights, a pervasive feature in many countries (Brown, 2000), is sustainable growth still possible? Third, is the long-run rate of economic growth affected by ill-defined property rights?

We consider a small and open economy which uses three assets to produce two final goods, one resource based and the other produced without using the resource. One asset is specific to the non-resource sector (“physical capital”), another is specific to the resource-based sector (natural capital) and the third asset (“human or knowledge capital”) enhances the productivity of labor in both sectors. We present an intuitive but novel explanation for sustainable growth: endogenous structural change. Structural change refers to a reduction of output and employment shares of the primary or natural resource sector during the growth process, a widely accepted stylized fact of the modern growth process (Chenery, 1960; Kuznets, 1957; Kongsamut et al. , 2001).

First we derive the conditions for sustainable development when property rights are well-defined on all assets including the natural resource, and markets are competitive using the “benevolent social planner” paradigm. Next we consider the case where the natural resource is subject to ill-defined property rights but property rights are well defined for the other two assets and markets are competitive. In both cases the model predicts a decreasing share of the primary (resource-based) sector’s output in total output, a decreasing share of the total labor force employed in the primary sector, and unbalanced asset accumulation causing a continuous change in the composition of factors of production toward those most intensively used by the non-primary sectors. The model thus predicts a broader version of structural change than usual. We also find

that labor productivity grows in both the resource and non-resources sectors. These predictions seem consistent with well accepted stylized facts<sup>1</sup>.

We also show that property rights on the natural resource are neither necessary nor sufficient for sustainable economic growth. Lack of property rights cause a lower equilibrium level of the natural resource (and a lower welfare level at each point in time) but their absence threatens neither the feasibility nor the actual *rate* of economic growth over the long run. That is, the equilibrium rate of economic growth is unrelated to the level of natural resource endowment; resource-rich countries may not grow any faster over the long run than resource-poor ones.

Intuitively, structural change is an endogenous response to the nature's constraint which implies that the natural resource remains constant over the long run. Even if technical change has identical impacts across sectors, growth in the productivity of man-made assets in the primary sector cannot match the growth in productivity in the non-resource sector where all assets continuously expand over time. This implies that the non-resource sector is able to continuously pull labor from the primary sector and to grow faster than the primary sector. Under open access, there is no investment in the natural asset, and the natural resource is over-harvested. However, despite these problems, labor market interactions preserve structural change and the level of the natural resource also remains constant over the long run, albeit at a lower level than under perfect property rights. Given well functioning markets elsewhere, increases in man-made capital still imply a divergence of productivity growth across the two sectors. To maintain equilibrium in the labor market, labor is drawn towards the non-resource sector.

The result that optimal environmental policy is not necessary for sustainable growth is in sharp contrast with the findings by the literature focusing on pollution rather than renewable natural resources, in which optimal environmental policy is a necessary condition for sustainable growth (Stokey, 1998; Aghion and Howitt, 1998). The reason is that this literature assumes that pollution

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<sup>1</sup> See, for example, Parry, 1999 for empirical evidence regarding the latter prediction.

only affects the utility function but it has no feed-back (negative) effects on the productivity of the production sector. Thus, in the absence of environmental policy that imposes a cost to pollution emissions, firms have no incentives to restrain emissions; sustainable growth is, therefore, infeasible. By contrast, when the environment is also a factor of production, as the natural resource becomes scarcer firms have an incentive to substitute and to shift toward non resource using sectors.

The literature on sustainable growth is, with a few exceptions, based on models with a single final good where the growth of income and environmental damage derive from the same sector<sup>2</sup>. Often this implies that sustained and environmentally sustainable growth is possible only after making special assumptions. Smulders and Gradus (1996) assume a unitary elasticity of substitution between pollution and physical capital in production and that abatement expenditures are relatively more effective in reducing pollution than increases in pollution from growth.<sup>3</sup> Similarly, Huang and Cai (1994) assume the presence of government spending externalities on pollution control, and Schou (2000) posits human capital externalities as sources of sustainable growth.

Stokey (1998) proposes exogenous technical change as the reason for sustained and environmentally sustainable growth. Aghion and Howitt (1998) show that sustainable growth is possible without exogenous technical change, or exogenously imposed externalities. However, they remain concerned with the assumption that the elasticity of marginal utility of consumption has to be greater than one (or that utility is highly concave), which is necessary in both Stokey's and Aghion and Howitt's models to attain sustainable growth.<sup>4</sup> This assumption ensures that individuals are willing to make the required sacrifices in consumption needed for sustainability. The fact that we focus on natural resources as a factor of production means that in sharp contrast with their analyses,

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<sup>2</sup> See Brock and Taylor (2004), and Smulders (1999) for excellent surveys.

<sup>3</sup> Similar to Smulders and Gradus (1996) we assume a unitary elasticity of substitution between natural and physical capital, however, we do not assume that investment in natural capital (equivalent to pollution abatement) is more effective in expanding the natural resource than harvest (or pollution) is in reducing such resource.

<sup>4</sup> "It appears that unlimited growth can indeed be sustained, but it is not guaranteed by the usual sorts of assumptions that are made in endogenous growth theory" (Aghion and Howitt, 1998, p. 162).

we neither need optimal environmental policy nor to impose the assumption of a greater than unitary elasticity of the marginal utility as necessary conditions for sustainable development.<sup>5</sup>

A recent paper by Eliasson and Turnovsky (2004) is concerned with sustainable and endogenous growth in a two-sector small open economy with the renewable natural resources as a factor of production. Our paper differs from Eliasson and Turnovsky in several respects. Firstly, we do not rely on capital accumulation externalities as they do to induce productivity growth. Productivity in our model grows due to endogenous increases in human or knowledge capital. Secondly, while there is structural change in their model, it is limited to changes in the composition of the value of output. There is no structural change in the composition of inputs employed. More importantly, structural change in their model is the result of the assumption that productivity grows only in the non-resource industry (due to the presence of an externality) while any productivity growth in the resource sector is *a priori* ruled out. In contrast, in our model structural change is an endogenous response brought about due to an unbalanced growth in assets and labor market interactions despite that labor-augmenting technical change takes place at the same rate in both the resource and non-resource sectors.

McAusland (2003) also relies on learning-by-doing externalities to generate economic growth in a model with two factors of production. Further, she focuses exclusively on the case of open access, and then studies the possibility of sustainable growth in both a closed and open economy. In contrast, we consider the open economy case, but study sustainable growth under well-defined property rights, and under ill-defined property rights or open access.

The present paper is also related to the literature explaining structural change. In this literature structural change is the result of non-homothetic preferences (Echevarria (1998), Laitner (2000) and Kongsamut et al. (2001)) or of imbalanced technical change (Baumol (1967) and Baumol

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<sup>5</sup> Intuitively, conditions sufficient for sustainable growth in the absence of utility benefits are also likely to be sufficient for sustainable growth when the resource produces utility in addition to output. If maintaining a stable stock of natural

et al. (1985). As consumers place a higher value on certain types of goods on becoming wealthier, the economy changes its demand toward non-resource goods. Also, if technical change is sector biased toward the non-resource sector structural change will take place. By contrast, we show that structural change is likely to take place even if preferences are homothetic and technical change is balanced. Given a constraint on the size of the natural asset, a relatively well functioning economy adjusts assets, and subsequently outputs, biasing growth towards the non-resource sector.

The paper is organized as follows. Section II presents the model, Section III defines and characterizes the long-run equilibrium, Section IV examines the consequences of the absence of an optimal environmental policy, and Section V concludes.

## II. The Model

Consider a small open economy with a representative agent consuming two final goods. One of the goods, the *clean* one, has no impact on the environment, while production of the *dirty* good, representing all natural resource- intensive sectors of the economy, harms the environment. There are three assets, human or knowledge capital, natural capital, and physical capital.

### Consumption

Let  $x_c$  denote consumption of the clean good and  $x_d$  the consumption of the dirty good.

Preferences of the representative consumer are given by,

$$U(x_c, x_d) = \frac{\varepsilon}{\varepsilon - 1} [x_c^\xi x_d^\psi]^{\frac{\varepsilon - 1}{\varepsilon}}, \quad (1)$$

where,  $\varepsilon > 0$  is a fixed parameter representing the elasticity of inter-temporal substitution or the reciprocal of the elasticity of marginal utility, and  $\xi + \psi = 1$ .

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resources while preventing extinction is possible with only benefits in output, it is also likely to be possible when consumers in addition directly value natural resources.

## Production and Labor Market Clearing

Production in the clean sector uses raw labor ( $l_c$ ), labor augmenting knowledge or human capital ( $h \geq 1$ ), and sector-specific non-human or “physical” capital ( $k$ ). The production function is

$$y_c = F(k, hl_c), \quad (2)$$

where  $F(\cdot)$  is concave, increasing and homogenous of degree one in  $k$  and  $hl_c$ , and satisfies the *Inada conditions* (as an input approaches zero its marginal product tends to infinity).

Production of the dirty (or resource dependent) good uses raw labor ( $l_d$ ) and the stock of natural capital ( $n$ ) as inputs. This sector also benefits of labor-augmenting technical change,  $h$ . The production function for the dirty good is

$$y_d = nhl_d. \quad (3)$$

This function is the standard specification for production based on natural resources first introduced by Gordon (1954) and Schaefer (1957) for fisheries who argued that the law of diminishing returns was not appropriate in this case. Since then this specification has been widely used in modeling natural resources (Brander and Taylor (1998 and 1998a), Conrad (1995), and Munro and Scott (1993)). A convenient interpretation of Equation(3) is  $y_d = [\phi hl_d + (1 - \phi)hl_d]n$ , where  $\phi nhl_d$  is the resource harvest, and  $(1 - \phi)nhl_d$  is processing value-added. We assume that producing one unit of the dirty good reduces the natural capital by less than one unit,  $0 < \phi < 1$ .

The labor market is perfectly competitive. Labor market clearing requires the sum of labor employed in the dirty and clean sectors to equal the total labor supply ( $\bar{L}$ ),  $l_c + l_d = \bar{L}$ .

## Asset Accumulation

Let  $I_j$  denote investment in each of the assets  $j \in \{h, n, k\}$ . Growth in labor augmenting human (knowledge) capital is given by

$$\dot{h} = I_h - \delta_h h, \quad (4)$$

where  $\delta_h$  is the rate of depreciation of  $h$  and  $I_h \geq 0$ . The rate of depreciation of knowledge can be interpreted as the proportion of annual retirements from the workforce. The non-negativity constraint for  $I_h$  is quite natural, indicating that labor-augmenting knowledge can be expanded but not reduced except through natural depreciation over time.

Let  $g(n)$  be the intrinsic growth function of the renewable natural resource. The function has an inverted U shape with  $g(0) = g(\bar{n}) = 0$ , where  $\bar{n}$  is the ‘carrying capacity’ of the natural resource. The *carrying capacity* of a natural resource is the maximum stock that can be sustained in its natural surroundings.<sup>6</sup> The function is assumed to be concave.<sup>7</sup>

Let  $I_n$  denote investment in natural capital, such as tree planting, fish replenishment including aquaculture investments, protection or cleaning-up of ecosystems, soil protection including terracing, drainage, and agricultural fallowing. Evolution over time of the natural resource stock is

$$\dot{n} = g(n) + I_n - \phi n h_l_d, \text{ if } \bar{n} \geq n \geq 0. \quad (5)$$

Growth of the natural resource comprises its natural capacity to regenerate ( $g(n)$ ), investment in expanding natural capital ( $I_n$ ), and the reduction of natural capital from production of the dirty good ( $\phi n h_l_d$ )<sup>8</sup>. We assume that  $I_n \geq 0$ , efforts will not be spent in reducing the stock of the natural resource except through its more intense extraction. Also, investment cannot maintain the natural resource beyond its carrying capacity. Investment can substitute for natural regeneration only if the

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<sup>6</sup> If there is no extraction, the stock of natural resource stabilizes at its carrying capacity (see Conrad, 1995, for other examples of such a growth function).

<sup>7</sup> For example a logistic growth function where  $g(n) = \gamma n(1 - n/\bar{n})$  with  $\gamma$  being the maximum or *intrinsic* growth rate of the stock.

<sup>8</sup> Expression (5) can be generalized to allow for a non-linear effect of  $I_n$  on the growth of the stock,  $n$ :

$\dot{n} = g(n) + m(I_n) - \phi n h_l_d$ , where  $m(I_n)$  is an increasing and strictly concave function. This specification is more satisfactory because it allows the effectiveness of investment in natural resource conservation to fall as such investment increases. However, the main results are not affected when we assume that  $m(I_n)$  is linear instead (a proof can be requested from the authors). Thus, for expositional clarity we retain the linear specification for (5) the text.

natural resource is within its natural bounds. This reflects the fact that a natural resource involves constraints outside human control. If investment in the natural resource could maintain the stock beyond its natural carrying capacity, a natural resource would be no different from other forms of man-made capital.

The stock of physical capital  $k$  grows according to the following equation.

$$\dot{k} = I_k - \delta_k k, \quad (6)$$

where  $\delta_k$  is the rate of depreciation of  $k$  and  $I_k \geq 0$  is the investment in physical capital.

### The Social Planner's Problem

The social planner maximizes the present value of utility for the representative consumer by optimally investing in human, natural and physical capital. The social planner can be interpreted as a benign dictator who sets all variables to optimal levels conditional on exogenous variables. Alternatively, the planner's solution can be considered as the outcome of an economy where all markets function competitively with perfectly defined property rights for all assets<sup>9</sup>.

The social planner's maximization problem is

$$V \equiv \max_{x_c, x_d, l_c, I_h, I_n, I_k} \int_0^{\infty} U(x_c(t), x_d(t)) \exp^{-\rho t} dt, \quad (7)$$

where  $t$  denotes time and  $\rho$  is the social discount rate. Maximization of utility is subject to the following set of constraints: all capital growth equations ((4)-(6)), initial conditions for capital stocks, ( $h(0) = h_{00}$ ,  $n(0) = n_{00}$ , and  $k(0) = k_{00}$ ), non negativity constraints  $x_c(t) \geq 0$ ,  $x_d(t) \geq 0$ ,  $I_h(t) \geq 0$ ,  $I_n(t) \geq 0$ , and  $I_k(t) \geq 0$ , labor constraint  $0 \leq l_c \leq \bar{L}$ , and the budget constraint or current account equilibrium,

$$x_c + px_d + I_h + I_n + I_k \leq pnh[\bar{L} - l_c] + F(k, hl_c), \quad (8)$$

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<sup>9</sup> This equivalence has its roots in the first and second fundamental theorems of welfare economics (see Arrow and Debreu (1954) and the references included therein).

The relative price of the dirty good is  $p$  and the price of the clean good has been normalized to one. In a small and open economy  $p$  is exogenously given. The budget constraint requires that total consumption and investment expenditures should be no greater than society's total income.<sup>10</sup> If the budget constraint holds with equality (which occurs given our maximization assumption), it necessarily implies balanced trade and vice-versa (Dixit and Norman, 1980).

Let  $\lambda$  be the Lagrangian multiplier associated with the budget constraint, and  $\mu$ ,  $\eta$  and  $\Omega$  be the co-state variables associated with human, natural, and physical capital, respectively. The solution to the problem in equation (7) is found by maximizing a current value Hamiltonian,

$$\begin{aligned} \mathbf{H} = & U(x_c, x_d) + \lambda \left[ F(k, hl_c) + pnh(\bar{L} - l_c) - x_c - px_d - I_h - I_k - I_n \right] \\ & + \mu [I_h - \delta_h h] + \Omega [I_k - \delta_k k] + \eta \left[ I_n + g(n) - \phi nh(\bar{L} - l_c) \right] \end{aligned} \quad (9)$$

where  $\mathbf{H}$  is defined under the assumption that the natural resource is within its natural bounds  $n \in (0, \bar{n})$ . Later in Proposition 3 we present the conditions that ensure the equilibrium outcome satisfies the natural bounds.

The first order conditions with respect to the two consumption goods are,

$$\xi(x_c^\xi x_d^\psi)^{\frac{\varepsilon-1}{\varepsilon}} (x_c^{\xi-1} x_d^\psi) = \lambda, \quad (10)$$

$$\psi(x_c^\xi x_d^\psi)^{\frac{\varepsilon-1}{\varepsilon}} (x_c^\xi x_d^{\psi-1}) = \lambda p. \quad (11)$$

Using (10) and (11) it follows that at the optimum  $x_c$  and  $x_d$  are consumed in fixed proportion for a given level of  $p$ , and thus we can represent the consumption optimality condition in terms of the marginal utility of  $x_c$  only:

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<sup>10</sup> We assume that the clean good is used for investment (this is reflected in equation (9), where investments in assets are priced at unity). Note, however, that this does not imply that all revenues from the resource good are consumed. Such revenues can be used to import investment goods.

$$\zeta \left( \frac{\psi}{\xi p} \right)^{\frac{\psi(\varepsilon-1)}{\varepsilon}} x_c^{-\frac{1}{\varepsilon}} = \lambda \quad (12)$$

Since  $x_c$  and  $x_d$  are proportional for given  $p$ , we have that the rate of growth of U is entirely determined by the rate of growth of  $x_c$ .

The Kuhn-Tucker first order conditions with respect to investment in human, physical and natural capital are:

$$-\lambda + \mu \leq 0, I_h [-\lambda + \mu] = 0, I_h \geq 0, \quad (13)$$

$$-\lambda + \Omega \leq 0, I_k [-\lambda + \Omega] = 0, I_k \geq 0, \text{ and} \quad (14)$$

$$-\lambda + \eta \leq 0, I_n [-\lambda + \eta] = 0, I_n \geq 0. \quad (15)$$

The condition for the optimal allocation of labor between the two sectors is,

$$\lambda [F_2(\cdot)h - pnh] + \eta \phi nh = 0, \quad (16)$$

where  $F_2(k, hl_c) \equiv \partial F / \partial hl_c$ . Conditions (10) and (11) imply that the marginal values of consumption should equal the shadow value of income,  $\lambda$  (which is positive given positive marginal utilities). Kuhn-Tucker conditions (13) to (15) indicate that positive investment in any of the assets must imply that the shadow value of such asset is equal to the shadow value of income or consumption. If an asset's shadow value is below the shadow value of income there will be zero investment in such an asset. These conditions also imply that the assets' shadow value can be no greater than the marginal value of consumption,  $\lambda$ . If any of the assets shadow value is above  $\lambda$  it is optimal to instantaneously reduce consumption enough so that  $\lambda$  increases to the level of the asset shadow value and thus allow for additional savings to increase investment in such asset. A sufficient increase of the marginal utility of consumption to achieve this is always possible given that the Cobb-Douglas utility function satisfies the Inada conditions.

Condition (16) reflects efficiency in the labor market. At each point in time (whether in long-run equilibrium or not), the value of the marginal product of labor in the clean sector,  $\lambda F_2(\cdot)h$ , should equal the value of the marginal product of labor in the dirty sector,  $(\lambda pnh)$  net of the marginal resource degradation caused by labor in the resource sector,  $(\eta\phi nh)$ . The co-state variable dynamics for the three assets are,

$$\dot{\mu} = (\rho + \delta_h)\mu - \lambda(F_2(\cdot)l_c + pn(\bar{L} - l_c)) + \eta\phi n(\bar{L} - l_c) \quad (17)$$

$$\dot{\Omega} = (\rho + \delta_k)\Omega - \lambda F_1(\cdot) \quad (18)$$

$$\dot{\eta} = (\rho - g_n)\eta - h[\bar{L} - l_c](\lambda p - \eta\phi). \quad (19)$$

Finally the transversality conditions for this model are,

$$\lim_{t \rightarrow \infty} e^{-\rho t} \mu h = 0, \quad \lim_{t \rightarrow \infty} e^{-\rho t} \Omega k = 0, \quad \lim_{t \rightarrow \infty} e^{-\rho t} \eta n = 0. \quad (20)$$

Below we focus on a characterization of one type of long run equilibrium which is most relevant to the analysis of structural change (which obviously presumes a diversified economy). It can be shown that this equilibrium is locally stable and, moreover, that the economy will converge towards it from a broad range of combinations of the state variables that are not necessarily in the neighborhood of such equilibrium. A technical appendix showing the dynamic properties of the system is available from the authors.

### III.- Long Run Growth<sup>11</sup>

Differentiating (12) with respect to time we obtain the usual result that the rate of growth of  $x_c$  must be directly related to the negative of the rate of growth of  $\lambda$ ,

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<sup>11</sup> An explicit solution of the model with specific functional forms can also be requested from the authors. In this solution we use a Cobb-Douglas form for the production function of the clean good with  $F(k, hl_c) = Ak^\alpha (hl_c)^{1-\alpha}$  where  $A$  is a positive constant, and  $0 < \alpha < 1$  is the constant share of capital in total production. The natural resource growth function is logistic where  $g(n) = \gamma n(1 - n/\bar{n})$  with  $\gamma$  being the maximum or *intrinsic* growth rate of the stock.

$$\hat{x}_c = -\varepsilon \hat{\lambda}, \quad (21),$$

where we use a ‘hat’ to denote rate of change, i.e.  $\hat{x}_c \equiv \dot{x}_c / x_c$  (note that the proportionality of consumption implies that  $\hat{x}_c = \hat{x}_d$ ). If  $\hat{\lambda} < 0$  then any  $\varepsilon > 0$  is consistent with growing consumption. That is, growth does not require any restriction on the inter-temporal elasticity of substitution (and, consequently, on its reciprocal, the elasticity of the marginal utility) other than being positive.

We now define the sustainable growth equilibrium as,

**Definition.** We say that the planner’s problem achieves *long-run sustainable growth equilibrium* (LSGE) when the rate of growth of consumption is positive and constant over time and the level of the resource stock is constant, that is  $\dot{n} = 0$ .

This definition of LSGE is fairly general. Given a constant and positive rate of growth of consumption, the rate of growth of welfare is also constant and positive in LSGE. A positive growth in welfare combined with a constant stock of natural capital constitutes sustainable growth. We shall explore the implications of LSGE in Propositions 1 and 2 below. Proposition 3 presents the conditions under which the LSGE is feasible. Stability of LSGE is examined later.

**Proposition 1.** *In LSGE: (a)  $\lambda = \mu = \Omega > 0$ ; and (b)  $\lambda / \eta$  is constant. Consequently,  $\hat{\lambda} = \hat{\mu} = \hat{\Omega} = \hat{\eta}$ .*

*Proof:* See Appendix A.

Proposition 1 demonstrates that in LSGE all shadow values should decrease at the same rate and that the shadow values of consumption, human capital and physical capital should be identical. These results help us further characterize the solution for LSGE. We can rewrite the labor market equilibrium condition (equation (16)) to get,

$$F_2(k/hl_c, 1) = (p - r\phi)n \quad (22)$$

where  $r \equiv \eta / \lambda$  is a constant. Further, by setting  $\hat{\Omega}$  and  $\hat{\eta}$  equal to each other (using equations (18) and (19)) we obtain,

$$F_1(k / hl_c, 1) - \delta_k = g_n(n) + (p/r - \phi)h(\bar{L} - l_c). \quad (23)$$

The net marginal products of physical and natural capital are equal in LSGE. Next if we equalize  $\hat{\mu}$  and  $\hat{\Omega}$  (using (17), (18) and subsequently (22)) we obtain,

$$F_2(k / hl_c, 1)\bar{L} - \delta_h = F_1(k / hl_c, 1) - \delta_k. \quad (24)$$

The above equation implies that in the LSGE, net marginal products of  $k$  and  $hl_c$  in the clean sector equal each other. Note that given our production function, there is a unique  $(k / hl_c)$  level at which these net marginal products are equalized. In other words, this expression implies that the  $(k / hl_c)$  ratio remains constant during the LSGE.

Given a constant  $(k / hl_c)$  ratio, equation (22) implies that there is a unique and constant stock of natural capital,  $n^*$ , in LSGE. Together a constant stock of natural capital and a constant  $(k / hl_c)$  imply that  $h(\bar{L} - l_c)$  also remains constant in the equilibrium (see equation (23)). This means that  $k$  and  $h$  need to change at different (unbalanced) rates to preserve the constancy of  $h(\bar{L} - l_c)$ . Thus there is a unique relationship of  $h$  and  $k$  needed to maintain  $h(\bar{L} - l_c)$  at its constant long run level; this is the reason why  $h$  and  $k$  cannot change proportionally as in the standard case (more on this below).

In Figure 1 the horizontal line represents equation (24) while the solid upward-sloping line represents Equation (22), which is the equality among returns to labor between the two competing sectors. Since returns to labor in the clean sector are directly related to the level of natural resources, this latter line is positively sloped. The figure shows that if the level of natural resources  $n$  was

higher (lower) than  $n^*$ , the returns to human capital (in both sectors) would be higher (lower) than the returns to physical capital which would be unsustainable in LSGE.

The constancy of the stock of natural resources implies,

$$g(n)/n + I_n/n - \phi h(\bar{L} - l_c) = 0. \quad (25)$$

The system of equations (22) to (25) solves for the four endogenous variables:  $(k/hl_c)^*$ ,  $n^*$ ,  $[h(\bar{L} - l_c)]^*$ , and  $I_n^* \geq 0$ , or  $r^* \leq 1$ . In principle there are two possible solutions; the first is where  $I_n^* > 0$  and, therefore,  $r^* = 1$ . Alternatively we can have  $I_n^* = 0$ , and from (15),  $0 < r^* < 1$ . In Proposition 2 we demonstrate that there is positive investment in natural capital while at LSGE.

**Proposition 2.** *In LSGE  $r^* = \frac{\eta}{\lambda} = 1$  and, therefore,  $I_n^* > 0$ .*

*Proof.* See Appendix A.

Proposition 2 demonstrates that investment in natural capital is positive along the LSGE. This result also implies that the shadow value of natural capital equals marginal utility of consumption ( $\eta = \lambda$ ), and  $r^* = 1$ . Using this result, the rate of growth in consumption (from Equation (21)) along the LSGE can now be expressed as

$$\hat{x}_c = \varepsilon [F_2((k/hl_c)^*, 1)\bar{L} - \rho - \delta_h] \quad (26)$$

The next proposition provides the feasibility conditions necessary and sufficient for LSGE.

**Proposition 3.** *An economy may achieve LSGE if and only if: (a)  $\rho < F_2((k/hl_c)^*, 1)\bar{L} - \delta_h$ ; (b)  $\rho > [F_2((k/hl_c)^*, 1)\bar{L} - \delta_h](\varepsilon - 1)/\varepsilon$ ; (c)  $F_2((k/hl_c)^*, 1)\bar{L} - \delta_h \geq g_n(n^*) + [(p - \phi)/\phi]g(n^*)/n^*$ . In addition, LSGE may be **diversified** only if (d)  $p > \phi$ ; and (e)  $F_2((k/hl_c)^*, 1) < (p - \phi)\bar{n}$ .*

*Proof.* Please see discussion below.

Condition (a) ensures that the economy is productive enough for growth to be possible (see Equation (26)). The net marginal productivity of the assets must be greater than the discount rate

(this also implies that  $\hat{\lambda} = \hat{\mu} = \hat{\Omega} = \hat{\eta} < 0$ ). Condition (b) results from the transversality conditions (see Appendix B for a proof). If this condition does not hold the objective function is convex and welfare cannot be maximized.

Condition (c) is needed for Proposition 2 to hold, it provides the conditions for  $g(n^*)/n^* - \phi h(\bar{L} - l_c)^* \leq 0$ , which implies that in LSGE  $I_n^* \geq 0$  and  $\eta = \lambda$ , both of which are required for a diversified equilibrium. In addition, this condition assures that the net marginal product of man-made assets in the clean sector is at least as large as the net marginal product of nature in the primary sector (that is, that equation (23) holds). Thus, condition (c) ensures that the clean sector can compete with the primary sector. Because there is a unique equilibrium ratio,  $k/hl_c$ , the marginal products of the assets used by the clean sector are in fact constant. Hence, the Inada conditions are not sufficient to assure the survival of the clean sector. If the productivity of the clean sector is too low or if the price of the dirty good is too high LSGE is not feasible.<sup>12</sup>

If condition (c) is not satisfied the economy specializes in the dirty sector. Specialization in the dirty sector, however, precludes LSGE. The reason is that natural capital cannot expand beyond the maximum carrying capacity,  $\bar{n}$ . That is, at some point economic growth must come to a halt. This occurs because, in contrast with the case of a diversified equilibrium, the expansion of  $h$  along the growth process can no longer be compensated by a reduction of  $l_d$ . As  $h$  increases natural resource extraction continuously expands, which requires a continuous increase in investment in the natural asset. However, given a limit of carrying capacity, this is not possible.

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<sup>12</sup> To illustrate condition (c) consider the specific functional forms  $F(k, hl_c) = Ak^\alpha (hl_c)^{1-\alpha}$  and  $g(n) = \gamma n(1 - n/\bar{n})$  from footnote 11. Condition (c) under these functional forms reduces

to  $p \leq \frac{A\alpha^\alpha (1-\alpha)^{1-\alpha}}{\bar{n}} \left[ \frac{\phi}{\gamma} \bar{L}^{(1-\alpha)} + (2\phi+1)\bar{L}^{(1-\alpha)} \right] - \frac{\phi}{\gamma} \delta_h$ . It is more likely that condition (c) holds if  $A$ ,  $\bar{L}$  are high, and it is less likely that condition (c) holds if  $p$  is high.

Conditions (d) and (e) are necessary to ensure the survival of the primary sector but are not necessary for LSGE. If  $p < \phi$ , the revenues from producing one unit of the dirty good are less than the cost in environmental damage from production. In other words the sector is not competitive when property rights are well defined. However, even if  $p > \phi$  it is possible that the resource extractive sector is just not productive enough to be competitive (alternatively, the clean sector is comparatively too productive). Condition (e) rules this out. If this condition is not met, the highest possible returns to human capital in the dirty sector,  $(p - \phi)\bar{n}$ , are not enough to compete with the clean sector. In such a case, the dirty sector closes.

If, either condition (d) or (e) is not satisfied then the system specializes in the clean sector. In this case the model collapses into the well-known one sector, two-asset balanced growth model of Chapter 5 in Barro and Sala-I-Martin (1995) as a special case. That is, the standard two-asset growth model with constant returns to scale can be regarded as a characterization of an economy where the non-primary sector is so productive that the primary sector is unable to compete. However, conditions (d) and (e) are necessary but not sufficient for diversified long run equilibrium. As shown in the technical appendix (available from the authors), if the initial level of  $hl_d$  (which is, in fact, a state variable as its level at any point in time is entirely determined by  $h$ ,  $k$  and  $n$ ) is too high and/or the initial level of  $n$  is too low, the system may not converge to  $n^* > 0$  and, instead, follows a path towards extinction of the natural resource. In this case, of course, the long-run equilibrium is specialized in the non-resource sector and the solution is of the Barro and Sala-I-Martin type.

### **An Intuitive Discussion of the Sustainable Equilibrium**

Let us summarize the intuition underlying sustainability in the LSGE equilibrium with  $n^* > 0$ . We find that the clean sector grows while the dirty sector stagnates. This is because given that production in the clean sector derives solely from man-made assets it has the potential for greater growth. In order to maintain growth in this sector, positive investment in the two man-made assets

( $h$  and  $k$ ) is necessary. As the cost of investment in  $h$  or  $k$  in terms of consumption is equal ( $\lambda$ ) positive investment in both  $h$  and  $k$  takes place only when their net marginal value products are identical (this can be seen from the first order conditions that require their co-state shadow values to coincide). Given constant returns to scale in the clean sector, the net marginal products of  $h$  and  $k$  depend solely on the capital to effective labor ratio ( $k/hl_c$ ). This implies that there is a unique capital/effective labor ratio at which the net marginal products of both assets are equalized and constant over LSGE.

Given that the marginal product of effective labor is fixed over LSGE, the labor market ensures that the natural resource remains constant. Labor market equilibrium implies that labor is allocated across two sectors so as to equalize the marginal value products of effective labor in both sectors. As the marginal value product of effective labor in the dirty sector only depends on  $n$  it follows that the fixity of the marginal product of effective labor in the clean sector also forces a unique level of  $n$  compatible with such marginal product level. That is,  $n^*$  is fixed. Also, the fixity of the extraction effort ( $hl_d$ ) is needed. There is a unique level of  $hl_d$  at which the net marginal product of  $n$  equalizes those of  $k$  and  $h$ . A higher level of  $hl_d$  will induce incentives to invest more in natural resources and to stop investing in man-made assets. This would cause the net marginal product of  $n$  to fall and that of  $k$  to increase which, in turn, would cause a correction of the process back to the unique constant level of  $hl_d$  that guarantees equal returns to man-made and natural assets. The fixity of  $hl_d$  over the long run assures that  $n$  is also fixed; that is, economic growth is environmentally sustainable. Further, the constancy of  $hl_d$  and the fact that  $h$  continuously grow over the long run requires that  $l_d$  continuously falls.

### **Further Implications of the LSGE**

From the equilibrium level of  $k/hl_c$  derived from (24) we have that  $l_c = (k/h) \cdot c_1$ , where  $c_1$  is a positive constant that depends on the depreciation rates and parameters associated with the production function of the clean output. Using (22) and (23) (and using  $r=1$ ) in the above expression, we obtain the following equilibrium relationship between  $k$  and  $h$ :

$$k = \frac{h\bar{L}}{c_1} - \frac{\chi(n^*)}{(p-\phi)c_1}, \quad (27)$$

where  $\chi(n^*) \equiv (p-\phi)n^*\bar{L} - \delta_h - g_n(n^*)$ . Using equations (22) to (24) it immediately follows that  $\chi \geq 0$ . Equation (27) is depicted in Figure 2. Since both  $\chi(n^*)$  and  $c_1$  are positive, the equilibrium line crosses the vertical axis at negative levels of  $k$ . This means that  $\hat{k}$  must be larger than  $\hat{h}$ ; that is asset-unbalanced growth.

**Proposition 4.** *During LSGE the physical/human capital ratio,  $k/h$ , is increasing; that is,  $\hat{k} - \hat{h} > 0$ .*

*Proof.* See Appendix A.

**Corollary to Proposition 4.** *As  $t \rightarrow \infty$ ,  $\hat{h} \rightarrow \hat{k}$ .*

Along LSGE, the factor specific to the clean sector ( $k$ ) grows faster than the asset common to both sectors ( $h$ ) but their growth rates asymptotically converge. However, in finite time, the difference in growth rates has important consequences for the economy as shown by Proposition 5.

**Proposition 5.** *Along the equilibrium expansion path: (i) The share of labor employed in the dirty sector declines. (ii) The ratio of value of dirty output to the value of total output declines. (iii) The physical capital to output ratio in the economy rises.*

*Proof:* See Appendix A.

Parts (i) and (ii) of Proposition 5 provide predictions consistent with some of the most robust stylized facts of the modern economic growth process. Along the growth path, the share of employment in the primary (or dirty) sector falls, and that the share of output in the primary sector also declines (see, for example, Kongsamut et al. 2001). Most authors analyzing structural change,

attribute such changes to an evolution of consumer demand and/or biased technological change in the growth process. Here we show that structural change can occur even if demand plays no role (as is the case in a small open economy) and technological growth is unbiased (in the sense that it benefits both sectors in the same way). We show that structural change can occur solely because the primary sector is based on a natural resource with a finite carrying capacity.

**Corollary to Proposition 5.** *Labor productivity in the primary sector expands at a rate  $\hat{h}$ , the same rate at which productivity in the clean sector expands.*

*Proof:* See Appendix A.

The prediction from this corollary is also consistent with the experience of many countries.<sup>13</sup> However growth in output per worker has not prevented the fall in the primary sectors share in GDP. The reason has mostly been the migration of labor from the primary sectors.

As indicated earlier, it is well known that the social planner solution replicates decentralized market equilibrium if all assets have well-defined property rights, if all factor and product markets are competitive, and if agents have rational expectations (see, for example, Barro and Sala-I-Martin (1995), pp 71). However, the assumption that well-defined property rights exist for the natural resource might be unrealistic. This is especially true for developing countries. We now consider the case of ill-defined property rights without optimal environmental policy.

#### **IV.- Open Access to Natural Resources**

The economy completely ignores the long run implications of resource extraction and economic agents behave as if the shadow value  $\eta$  is zero. There is no investment in natural capital and the net marginal value products of man-made assets are not equalized to the net marginal contribution of natural capital (equation (23) ceases to hold). Under open access, equation (25) becomes,

$$(h(\bar{L} - l_c))^o = \frac{g(n^o)}{\phi n^o}, \quad (28)$$

where the superscript “o” denotes the solution under open access. Equation (22) also changes as the marginal product of labor in the dirty sector does not account for the fact that an extra unit of labor in the sector causes a loss of natural capital,

$$F_2((k/hl_c)^o, 1) = pn^o. \quad (29)$$

Equation (24) showing the equalization of the net marginal products of physical and human capital remains exactly the same. The following proposition characterizes open access equilibrium,

**Proposition 6.** *In long-run equilibrium under open access we have that: (i)  $(k/hl_c)^o = (k/hl_c)^*$ ; (ii)  $n^o = [(p - \phi) / p]n^* < n^*$ ; (iii) The rate of growth of the economy is not affected by the open access condition,  $\hat{x}_c^* = \hat{x}_c^o$  and  $\hat{x}_d^* = \hat{x}_d^o$*

*Proof:* See Appendix A.

Proposition 6 shows that sustainable development is possible even under open access and no environmental policy is in place. The rate of growth over the long-run is not affected by the environmental inefficiency. However, we find that the long run stationary level of the natural resources is lower under open access, than when property rights are well defined. Combining these two results it follows that the rate of economic growth is unrelated to the (equilibrium) level of natural resources.

What about the feasibility conditions for LSGE under open access? Clearly the conditions for positive growth and a well defined optimum, parts (a) and (b) of Proposition 3, are equally feasible under open access and under the benign planner solution. This follows directly from the fact that  $pn^o = (p - \phi)n^*$  as shown in Proposition 6. Conditions (c) and (d) from Proposition 3 are no longer necessary in open access. The lack of relevance of condition (c) means that the survival of the clean sector is now assured solely by the Inada conditions.

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<sup>13</sup> Faruqui et al. (2003), for example, estimate the annual labor productivity growth of the primary industries in the USA at 3.1% while that of the manufacturing sector at 3.3% per year.

Condition (e) in Proposition 3, is easier to satisfy under open access than under the social planner. This condition under open access becomes  $F_2((k/hl_c)^o, 1) < p\bar{n}$  instead of  $F_2((k/hl_c)^*, 1) < (p - \phi)\bar{n}$  under the planner's solution. The left hand sides in the above expressions are equal but under open access assuring the permanence of the dirty sector over the long-run requires a lower price level. Proposition 7 below summarizes these results,

**Proposition 7.** (i) *The feasibility conditions for the existence of positive growth over LSGE are not more stringent under open access to natural resources than under optimal natural resource policy; (ii) under open access it is more likely that the primary sector will be able to survive as a productive sector; (iii) Under open access, and in contrast with the case of well-defined property rights, the Inada conditions are sufficient to assure the survival of the clean sector.*

Thus lack of property rights on the natural resource does not affect the potential rate of growth of the economy and neither does it affect the likelihood that the economy achieves sustainable economic growth. Optimal environmental policy is neither a necessary nor a sufficient condition for LSGE. Equally important, there is no reason to expect that “resource-rich” countries will grow over the long-run any faster than resource-poor countries.

## V.- Conclusion

We have shown that structural change allows an open economy to grow at a constant rate with environmental sustainability. Endogenous structural change is a response to the presence of a constraint on natural capital which takes place even if consumer preferences are homothetic and if technological progress does not discriminate against the primary or dirty sector. In the long run growth equilibrium the primary sector remains stagnant while the non resource using sector grows continuously by attracting an increasing volume of increasingly productive labor from the primary sector.

Property rights on the natural resource (or optimal environmental policy) are neither necessary nor sufficient for environmentally sustainable economic growth. The rate of growth of

the economy over the long run is not affected by environmental policy and the feasibility of environmental sustainability with positive long-run growth is not hampered by incomplete property rights. Optimal environmental policy induces a higher equilibrium stock level of natural resources vis-à-vis the open access case.

The rate of economic growth is thus not affected by the (equilibrium) level of resource endowment. Resource-rich and resource-poor economies can grow at similar rates over the long run as long as their non-environmental policies are identical and their non-resource sectors are similar. Moreover, a resource poor economy (say, an economy that has imperfect resource property rights) can grow faster than a resource rich economy (say, an economy that has perfect property rights) if the latter has more appropriate non-environmental policies. The real determinant of the feasibility and speed of sustainable growth is the development of a sufficiently productive clean sector that can successfully compete with the primary sector for factors of production, and a sufficiently rapid pace of accumulation of knowledge that enables labor-augmenting productivity and structural change. Market failures that affect, for example, the ability of the economy to invest in knowledge, may frustrate in the long-run structural change and ultimately the capacity of the economy to achieve both positive growth and environmental sustainability.

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## Appendix

### A. Proofs

*Proof to Proposition 1.* For having permanent positive consumption growth, growth in at least one asset must be positive. Either investments in human capital, physical capital or both should be positive in LSGE (note that natural capital cannot grow for ever). We consider these two cases separately. The labor market equilibrium condition (16) which holds both in LSGE and out of equilibrium, can be rewritten as,

$$F_2(k/hl_c, 1) = (p - \phi\eta/\lambda)n \quad (A1)$$

Case (1): Positive accumulation of human capital,  $I_h > \delta_h$ . From the first order conditions this requires that  $\lambda = \mu$  which using (21) implies that

$$\hat{x}_c = -\varepsilon\hat{\mu}.$$

Thus a constant rate of growth of consumption implies that  $\hat{\mu}$  is constant over time. Using this and equations (A1) and (17) consumption growth is given by,

$$\hat{x}_c = \varepsilon \left( p - \frac{\eta}{\lambda} \phi n \bar{L} - \rho - \delta_h \right) \quad (A2)$$

Since, given the definition of LSGE, both  $\hat{x}_c$  and  $n$  are constant, the above equation implies that  $\eta/\lambda$  must also be constant. This implies that  $\hat{\lambda} = \hat{\mu} = \hat{\eta}$  is necessary for LSGE. Further, (A1) implies that  $(k/hl_c)$  must also be constant in LSGE. The fact that the rate of growth of  $x_c$  is constant by definition of LSGE implies that  $\hat{\mu}$  and, therefore,  $\hat{\eta}$  are both constant. Given the constancy of  $\eta/\mu = \eta/\lambda$  and the fact that the resource stock,  $n$ , is also constant, (19) implies that the variable  $h(\bar{L} - l_c)$  must also be constant over time. Differentiating we get that

$$\hat{h} = [l_c/(\bar{L} - l_c)] \cdot \hat{l}_c \quad (A3)$$

which is greater than zero by assumption. This means that both  $h$  and  $l_c$  are increasing over time. But since  $(k/hl_c)$  must be constant, we have that  $\hat{k} > 0$ , which in turn means that  $I_k > 0$  and, therefore,  $\Omega = \lambda$ . Hence, LSGE requires that  $\hat{\Omega} = \hat{\lambda} = \hat{\mu} = \hat{\eta}$ .

Case (2): Alternatively, if  $k$  is accumulated, then  $I_k > \delta_k$ , and we have that  $\lambda = \Omega$  and thus  $\hat{x}_c = -\varepsilon\hat{\Omega}$ . In this case a constant growth rate of  $x_c$  implies that  $\hat{\Omega}$  is constant and negative. From (18) it immediately follows that  $F_1(k/hl_c, 1)$  is constant which in turn implies that  $(k/hl_c)$  is also constant. This in turn, means that  $\eta/\lambda = \eta/\Omega$  does not change over time. Therefore,  $\hat{\Omega} = \hat{\eta}$ ,  $\hat{\Omega} = \hat{\mu}$ . Equalizing (18) and (19) makes it clear that as in case (1),  $h(\bar{L} - l_c)$  is also constant over time. Therefore, we also have that (A3) holds. Now using the latter expression and the fact that  $(k/hl_c)$  is constant which means that:  $\hat{k} = \hat{h} + \hat{l}_c$ , we obtain that  $\hat{k} = (1 + [(\bar{L} - l_c)/l_c]) \cdot \hat{h}$ . Given that  $\hat{k} > 0$ , we have that  $\hat{h} > 0$  and, therefore,  $I_h > 0$  which, in turn, means that  $\mu = \Omega = \lambda$  and, as in case (1),  $\hat{\Omega} = \hat{\lambda} = \hat{\mu} = \hat{\eta}$ . ■

*Proof to Proposition 2.* Suppose that the non-negativity constraint is binding. That is,  $I_n = 0$  while in fact the unrestricted optimum would require  $I_n^* < 0$ . Use a tilde to define the solution of the system

(22) to (25) when  $I_n = 0$  and a double star to denote an hypothetical solution that allows for an unrestricted optimum,  $I_n^{**}$ , which is less or equal to zero if  $I_n = 0$ . We now compare the restricted and unrestricted (hypothetical) solutions. In this case condition (25) is satisfied if  $\tilde{n} < n^{**}$  or  $\overline{hl}_d < (hl_d)^{**}$ , or both (where  $l_d = \bar{L} - l_c$ ). If  $\tilde{n} < n^{**}$  then by (22)  $\tilde{r} < r^{**} = 1$ . Also, from (23) it is clear that since  $g_n(n)$  is decreasing in  $n$  that  $\overline{hl}_d < (hl_d)^{**}$ . Using analogous reasoning, it is clear that if  $\overline{hl}_d < (hl_d)^{**}$  then  $\tilde{n} < n^{**}$  as well. That is, we need both inequalities to hold simultaneously.

We now show that these two inequalities together are inconsistent with the first order conditions when evaluated at LSGE. Assume that  $\tilde{n} < n^{**}$  and  $\overline{hl}_d < (hl_d)^{**}$ . The solution when  $I_n = 0$  implies

$$\overline{hl}_d = g(\tilde{n}) / \phi \tilde{n} \quad (\text{A4})$$

Also, since  $I_n^{**} < 0$  by assumption we can define a variable  $v = -I_n^{**} / (\phi n^{**}) > 0$  such that,

$$(hl_d)^{**} + v = g(n^{**}) / (\phi n^{**}) \quad (\text{A5})$$

Given that  $g(n)/n$  is decreasing in  $n$ , we have that  $g(n^{**}) / (\phi n^{**}) < g(\tilde{n}) / (\phi \tilde{n})$ . Now subtracting (A5) from (A4) we obtain,

$$\overline{hl}_d - hl_d^{**} = g(\tilde{n}) / (\phi \tilde{n}) - g(n^{**}) / (\phi n^{**}) + v > 0,$$

which means that  $\overline{hl}_d > hl_d^{**}$  thus contradicting the earlier supposition. Hence either  $\tilde{n} > n^{**}$  or  $\overline{hl}_d > hl_d^{**}$ . But  $\tilde{n} > n^{**}$  means that  $\tilde{r} > r^{**} = 1$  according to (22), which is inconsistent with the Kuhn-Tucker condition (15). The other option,  $\overline{hl}_d > hl_d^{**}$  is not consistent with the condition (23) when  $\tilde{n} \leq n^{**}$ . Hence, the only feasible equilibrium is one with  $I_n^{**} > 0$  and  $r = 1$ . ■

*Proof to Proposition 4.* Taking time derivatives of (27):

$$\hat{h} = \hat{k} \left[ 1 - \frac{\chi(n^*)}{(p - \phi)h\bar{L}} \right]$$

which given that  $0 \leq \chi \leq (p - \phi)h\bar{L}$  then  $\hat{k} \geq \hat{h}$ , and that as  $h$  grows, so does  $k$ . ■

*Proof to Proposition 5.* The first two parts of Proposition 5 are derived from the fact that the total extraction effort  $(hl_d)^*$  as well as the level of  $n$  remain constant along the equilibrium growth path while the clean sector continuously grow as  $k$ ,  $h$  and  $l_c$  all expand. This leads to (ii). The fact that  $hl_d$  is constant while  $h$  continuously grows in LSGE means that  $l_d$  must decline over time. Hence, result (i) follows. The fact that the physical capital to output ratio is rising in the economy is not obvious but may be shown by looking at the definition:

$$\frac{k}{y} = \frac{k}{pnh(\bar{L} - l_c) + F(k, hl_c)} = \frac{1}{(h/k)pn(\bar{L} - l_c) + F_1(\cdot) + (h/k)l_c F_2(\cdot)},$$

where in the second equality we divided by  $k$  and expanded clean output using the property of homogeneity of the clean technology. If we replace  $l_c = (k/h) \cdot c_1$ , we get:

$$\frac{k}{y} = \frac{1}{(h/k)pn\bar{L} - pnc_1 + F_1(\cdot) + c_1 F_2(\cdot)}. \quad \blacksquare$$

*Proof to Corollary to Proposition 5.* Output per worker in the primary sector is  $hn$ . Since along LSGE  $n$  is constant at  $n^*$ , labor productivity increases at a rate  $\hat{h}$ . Output per worker in the dirty sector is  $F(k, hl_c)/l_c = F(k/l_c, h)$ . Where the equality follows from the fact that  $F$  is linearly homogenous in  $k$  and  $hl_c$ . Since  $F(k/l_c, h)$  is homogenous of degree one in  $k/l_c$  and  $h$  we have that  $F(k/l_c, h) = h F(k/hl_c, 1)$ . Noting that  $k/hl_c$  is constant along LSGE, we have that output per worker in the clean sector also grows at a rate  $\hat{h}$ . ■

*Proof to Proposition 6.* Part (i) follows from the fact that condition (24) is valid in open access as well. Under open access the shadow values of  $h$  and  $k$  are also equal and, therefore, they must decline at an identical and constant rate,  $\hat{\mu} = \hat{\Omega}$ . This means that  $F_1 - \delta_k = F_2 l_c - \delta_h + n(\bar{L} - l_c)$ . Using (28) in the above expression we obtain a condition identical to (24). That is,  $(k/hl_c)^o = (k/hl_c)^*$ . Part (ii) follows by noting that  $F_2((k/hl_c)^*, 1) = F_2((k/hl_c)^o, 1)$  (as equation (24) remains the same) and  $\phi > 0$ . From (22) and (29) we have that  $pn^o = (p - \phi)n^*$  which shows part (ii). Part (iii) is shown by using (21) noting that  $\hat{\lambda} = \hat{\mu} = \hat{\Omega}$ . Thus,

$$\hat{x}_c = -\varepsilon \hat{\lambda} = \varepsilon (F_2((k/hl_c)^o, 1)\bar{L} - \delta_h - \rho) = \varepsilon (F_2((k/hl_c)^*, 1)\bar{L} - \delta_h - \rho). \blacksquare$$

## B. Conditions Imposed by the Transversality Condition

In order to check the transversality condition, we need some idea about how assets are accumulated towards the end of the planning horizon, i.e. infinity. To get an approximation, we begin by taking a time derivative of the budget constraint (8),

$$\hat{x}_c x_c + \hat{x}_d p x_d = p n h l_d (\hat{h} l_d) + k F_1(\cdot) \hat{k} + h l_c F_2(\cdot) (\hat{h} l_c) - I_n \hat{I}_n - I_k \hat{I}_k - I_h \hat{I}_h \quad (A6).$$

Since the  $(k/hl_c)$  is constant in steady state, we have that  $\hat{k} = (\hat{h} l_c)$ , also recall that both  $I_n$  and  $h l_d$  are also constant in equilibrium, and that both consumption goods grow at the same rate. Furthermore, from Proposition 3 we know that at infinity both  $k$  and  $h$  grow at the same rate, therefore both investment rates must grow at the same rate at infinity, and that rate must be equal to the rate of growth of man-made assets. Thus, using the fact that the clean technology is homogenous of degree 1 we can rewrite (A6):

$$\lim_{t \rightarrow \infty} : \hat{x}_c (x_c + p x_d) = \hat{k} (F(k, hl_c) - I_k - I_h) \quad (A7).$$

Also, towards infinity we have that:

$$\lim_{t \rightarrow \infty} : \frac{x_c + p x_d}{F(k, hl_c) - I_k - I_h} \approx 1,$$

because the difference between numerator and denominator is a constant,  $p n h l_d - I_n$ , and its ratio over  $(F(k, hl_c) - I_k - I_h)$  tends to zero as clean output grows. Thus, we have that at infinity:

$$\lim_{t \rightarrow \infty} : \hat{x}_c \approx \hat{k} \equiv \omega(n^*) = \varepsilon [(p - \phi)n^* \bar{L} - \delta_h - \rho] \quad (A8).$$

The right hand side of (A8) contains the rate of growth of  $x_c$  from (21).

On the other hand, the rate of fall of  $\Omega$  may be readily obtained from (18):

$$\Omega(t) = \Omega_0 e^{-[(p-\phi)n^*\bar{L}-\delta_h-\rho](t-t_0)} \quad (A9),$$

where  $\Omega_0$  is the level of the shadow values when steady state was achieved (at time  $t_0$ ), and where we also used the fact that  $\hat{\Omega} = \hat{\mu}$ . We can now express the transversality condition for  $k$ :

$$\lim_{t \rightarrow \infty} \exp[-\rho t] \cdot k_\infty \exp\left[\varepsilon \left[ (p-\phi)n^*\bar{L} - \delta_h - \rho \right] t\right] \Omega_0 \exp\left[-\left[ (p-\phi)n\bar{L} - \delta_h - \rho \right] (t-t_0)\right] = 0 \quad (A10),$$

where  $k_\infty$  is a level of physical capital accumulated when time approaches infinity. From (A10), it can be shown that this condition will hold when,

$$\rho > (\varepsilon - 1) \left[ (p-\phi)n\bar{L} - \delta_h - \rho \right] \quad (A11).$$

Note that the right hand side of (A11) is equal to  $\hat{U}$ , so condition (A11) is similar to most endogenous growth models. Also note that (A11) can be re arranged like in condition (b) of Proposition 3. Given that both man-made assets grow at the same rate towards infinity, it is easy to show that condition (A11) also applies also for  $h$ . On the other hand, the transversality condition always holds for natural capital, as this asset is fixed in equilibrium, as long as  $\hat{\lambda} < 0$ .

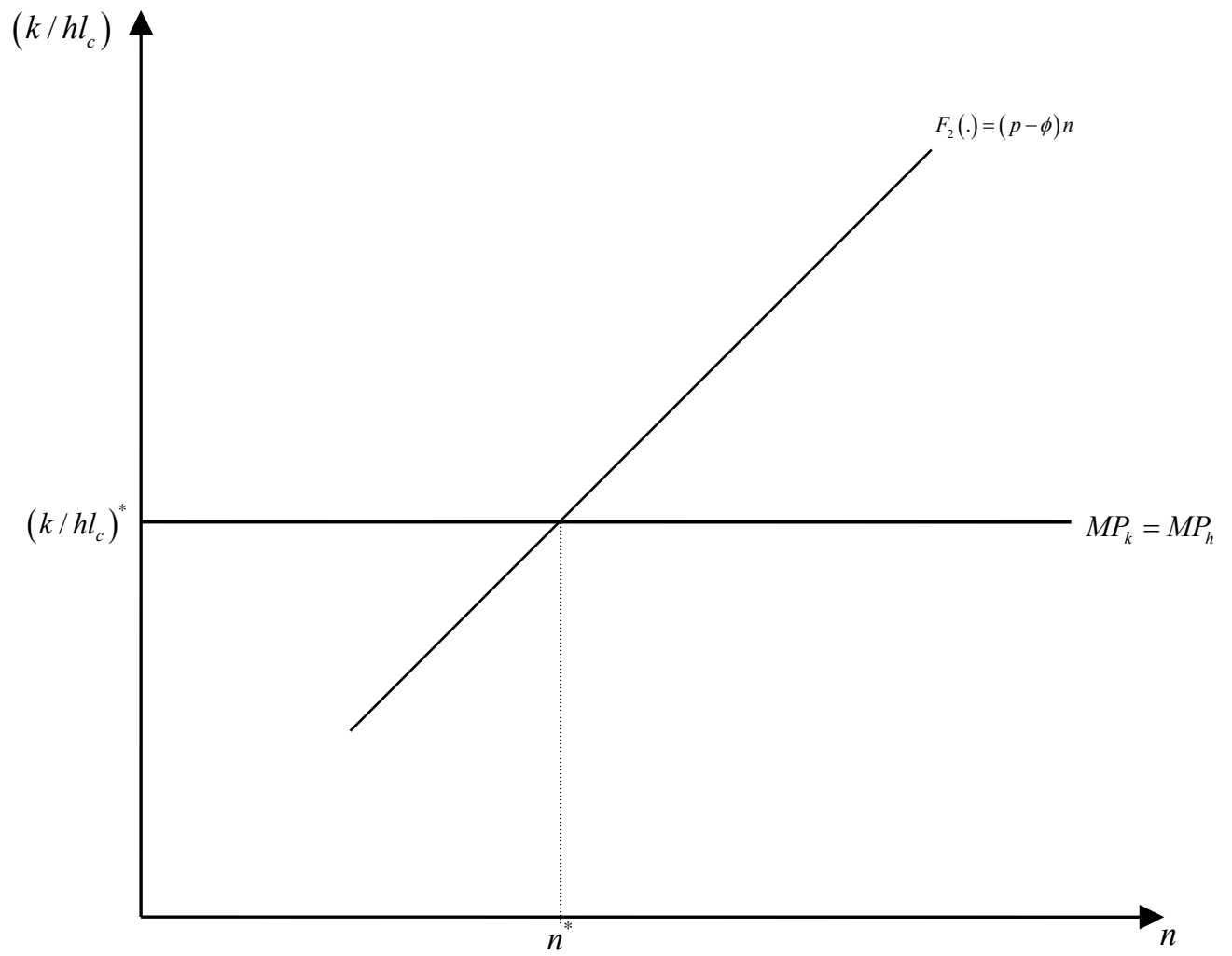


Figure 1

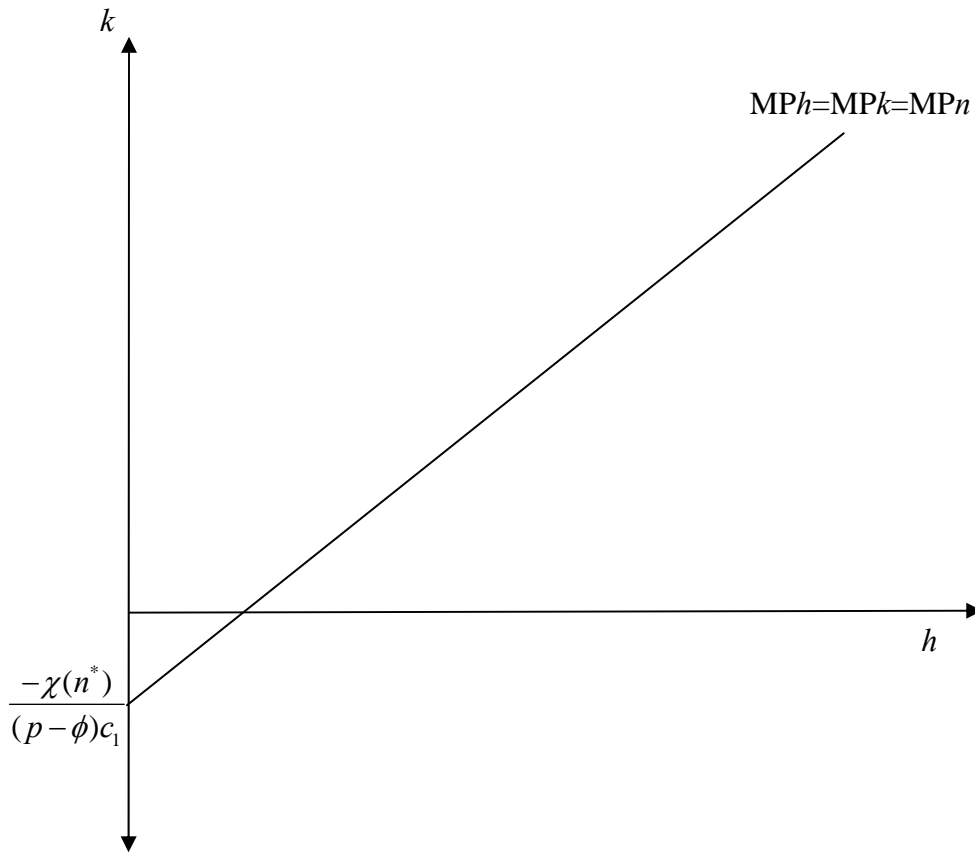


Figure 2